04 Assume that the magnetostatic equations hold on the infinite cylinder $\Omega \times \mathbb{R}$, where $\Omega \subset \mathbb{R}^2$ is bounded and simply connected, and assume further that

$$\boldsymbol{J} = \begin{pmatrix} 0 \\ 0 \\ J_3(x_1, x_2) \end{pmatrix}, \qquad \boldsymbol{H} = \begin{pmatrix} H_1(x_1, x_2) \\ H_2(x_1, x_2) \\ 0 \end{pmatrix}.$$

Verify that the choice

$$\boldsymbol{A} = \begin{pmatrix} 0 \\ 0 \\ A_3(x_1, x_2) \end{pmatrix}$$

for the vector potential implies the Colomb gauge, and that the magnetostatic equations (in the cylinder) simplify to

$$-\operatorname{div}\big(\frac{1}{\mu}\nabla A_3\big) = J_3 \qquad \text{in } \Omega, \tag{2.1}$$

where ∇ denotes the two-dimensional gradient.

05 Continue the previous exercise: let $\vec{n} = (n_1, n_2)^\top$ be the unit normal on $\partial\Omega$ or on any interface $\Gamma \subset \Omega$ and let $\boldsymbol{n} = (n_1, n_2, 0)^\top$ denote the corresponding normal on/in the cylinder.

(a) Show that

$$\boldsymbol{H} \times \boldsymbol{n} = \begin{pmatrix} 0 \\ 0 \\ J_{S,i,3} \end{pmatrix} \qquad \Longleftrightarrow \qquad \frac{1}{\mu}\nabla A_3 \cdot \vec{n} = J_{S,i,3}\,. \tag{2.2}$$

(b) Show that

$$A_3 \text{ continuous across } \Gamma \qquad \Longrightarrow \qquad [\boldsymbol{B} \cdot \boldsymbol{n}] = 0 \quad \text{on } \Gamma \times \mathbb{R}. \tag{2.3}$$

06 Consider a *plane wave*

$$\boldsymbol{E}(\boldsymbol{x}, t) = \boldsymbol{a}\, e^{i(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t)}$$

where $\boldsymbol{a} \in \mathbb{R}^3$ is the *amplitude*, ω the *angular frequency*, and $\boldsymbol{k} \in \mathbb{R}^3$ the *wave vector* (telling about the direction and the *wave number* $|\boldsymbol{k}|$). Assume that ε and μ are positive constants, and that $\sigma = 0$ (e.g. vacuum).

Find the conditions such that the plane wave fulfill equation (15) from the lecture, i.e.,

$$\varepsilon\,\frac{\partial^2 \boldsymbol{E}}{\partial t^2} + \sigma\,\frac{\partial \boldsymbol{E}}{\partial t} + \mathbf{curl}\,\frac{1}{\mu}\mathbf{curl}\,\boldsymbol{E} = 0. \tag{2.4}$$