---

01 Let us assume that $\boldsymbol{E}, \boldsymbol{D}, \boldsymbol{H}, \boldsymbol{B}$ are sufficiently smooth solutions of Maxwell's equations. Show that

(a) $\dfrac{\partial \rho}{\partial t} = -\operatorname{div} \boldsymbol{J}$,

(b) $\operatorname{div} \boldsymbol{B}(x, 0) = 0 \quad \Longrightarrow \quad \operatorname{div} \boldsymbol{B}(x, t) = 0$ for all $t > 0$.

Hint: use Ampére's law and Faraday's law.

02 Let $V \subset \mathbb{R}^3$ be partioned into two disjoint subdomains V_1 and V_2, i.e. $\overline{V} = \overline{V}_1 \cup \overline{V}_2$, $V_1 \cap V_2 = \emptyset$. Let $\Gamma = \overline{V}_1 \cap \overline{V}_2$ denote the interface between the subdomains. Let us assume the linear material law $\boldsymbol{B} = \mu \boldsymbol{H}$ in $\overline{V}$. Let $\boldsymbol{H}_i$, μ_i be the restrictions of $\boldsymbol{H}$ and μ to V_i, respectively. Assume that μ_1, μ_2 are *different* constants, and that $\boldsymbol{H}_2 \cdot \boldsymbol{n} \neq 0$ on Γ. Show that

$$[\boldsymbol{H} \cdot \boldsymbol{n}] = (\boldsymbol{H}_2 - \boldsymbol{H}_1) \cdot \boldsymbol{n} \neq 0 \quad \text{on } \Gamma.$$

03 Show the following interface condition for the $\boldsymbol{E}$-field:

$$[\boldsymbol{E} \times \boldsymbol{n}] = 0 \quad \text{on } \Gamma,$$

i.e. the tangential component of the $\boldsymbol{E}$-field is continuous across Γ.
Hint: Start from Faraday's law in integral form.