T U T O R I A L

"Numerical Methods for the Solution of Elliptic Partial Differential Equations"

to the lecture

"Numerics of Elliptic Problems"

Tutorial 14 Tuesday, 28 June 2011, Time: $10^{\underline{15}}$ – $11^{\underline{45}}$, Room: SR / T 642.

Programming (continued)

Incorporating boundary conditions (continued)

Let's consider Robin boundary conditions of the type

$$\frac{\partial u}{\partial N} := \lambda \frac{\partial u}{\partial n} = \kappa(u_0 - u) = g_3 - \kappa u.$$

for given λ, κ and u_0 and the normal derivative n.

67 Let $e \subset \Gamma_R$ be element edges on the Robin boundary with the two endpoints $x^{(e,1)}$ and $x^{(e,2)}$. Let the reference edge be $\Delta = (0,1)$ with the corresponding nodal basis functions $p^{(0)}(\xi) = 1 - \xi$ and $p^{(1)}(\xi) = \xi$. Write a function

```
void calcRobinElMat (const Vec<2>& x0, const Vec<2>& x1,
                     ScalarField kappa, Mat<2, 2>& elMat);
```

that computes the element Robin matrix K

$$K^e_{\alpha\beta} = \int_e \kappa(x) p^{(e,\alpha)}(x) p^{(e,\beta)}(x) dx = \int_\Delta \kappa(x_e(\xi)) p^{(\alpha)}(\xi) p^{(\beta)}(\xi) \det(J_e) d\xi$$

using the quadrature rule on $\Delta = (0,1)$ given by

$$\int_\Delta g(\xi) d\xi \approx \frac{1}{6}\left[g(0) + 4g(0.5) + g(1)\right].$$

Show that this quadrature rule is exact for $g \in P_3$.

Hint: In order to get $x_e(\xi)$, implement a class modelling the affine linear transformation for edges , i.e. in 1D (compare **33**, **34** and NumPDE-Tutorial).

68 Write a function

```
void incorporateRobinBC (const Mesh& mesh, ScalarField kappa,
                         ScalarField u0, SparseMatrix& K, Vector& b);
```

that incorporates the Robin boundary conditions into the system matrix `K` and the load vector `b`.

Hint: Loop over all segments of the mesh and search for those marked as Robin (use `bcSegments[i] == BC_ROBIN`) and reuse the function from the previous Exercise 67 to add the local contributions to the stiffness matrix.

Hint: For the contribution corresponding to $\int_{\Gamma_R} g_3(x)\, v(x)\, ds$, proceed as for the Neumann Boundary (see 52).

The CHIP-Problem

Recall the CHIP-Problem from the lecture (T08a, T08b, T09)!

69 Prepare the initial mesh for the CHIP problem as proposed on T09 in your mesh-format, taking care of the appropriate boundary conditions.

Hint: If possible use symmetric reduction.

70 Solve the finite element system corresponding to the CHIP problem on T08a with the parameter setting of T08b for the initial mesh of 69. Solve the same system for uniformly refined meshes with $h/h_0 = 2, 3, 8, 16$ and visualize the solution.

Hint: For incorporating the BC, use the following order: First natural BC, than essential BC.

A posteriori error estimates

71* Implement the residual error estimator for the CHIP-problem as derived in Exercise 66.

72* Compute the residual error for the CHIP-problem for uniformly refined meshes with $h/h_0 = 2, 3, 8, 16$ and visualize the error on each element!