

T U T O R I A L

“Numerical Methods for the Solution of Elliptic Partial Differential Equations”

to the lecture

“Numerics of Elliptic Problems”

Tutorial 10

Tuesday, 24 May 2011, Time: 10¹⁵ – 11⁴⁵, Room: SR / T 642.

3.6 Discretization Error Estimates

46 Show that for $d = 1$: $\Omega = (0, 1)$, $k = 1$: $\mathcal{F}(\Delta) = \mathcal{P}_1(\Delta)$, and $u(x) = x^2$ there holds

$$\inf_{v_h \in V_h} \int_0^1 |u'(x) - v_h'(x)|^2 dx = \frac{1}{3} h^2, \quad (3.29)$$

where $V_h = \text{span}\{p^{(i)} : i = 0, 1, \dots, n\}$ is defined using continuous affine linear finite elements on the mesh $0 = x^{(0)} < \dots < x^{(i)} = ih < \dots < x^{(n)} = 1$, $h = 1/n$.

47 Under the assumptions 1 and 2 of the Approximation Theorem 3.6, prove the completeness of the FE-spaces $\{V_h\}_{h \in \Theta}$ in $V = H^1(\Omega)$, i.e.,

$$\lim_{h \rightarrow 0} \inf_{v_h \in V_h} \|u - v_h\| = 0 \quad \forall u \in V. \quad (3.30)$$

3.7 Inverse-Inequalities

48 Compute the constant $c_A(\Delta)$ in the inequality

$$\max_{\xi \in \bar{\Delta}} \left| \sum_{\alpha \in A} v^{(r,\alpha)} p^{(\alpha)}(\xi) \right| \leq c_A(\Delta) \left\| \sum_{\alpha \in A} v^{(r,\alpha)} p^{(\alpha)}(\xi) \right\|_{L_2(\Delta)}, \quad (3.31)$$

used in the proof of Lemma 3.11, for linear triangular elements ($d = 2$, $k = 1$, $\mathcal{F}(\Delta) = \mathcal{P}_1$) !

49 Under the assumptions of Lemma 3.11, prove the inverse inequality

$$\|v_h\|_{L_\infty(\Omega)} \leq ch^{-\frac{d}{p}} \|v_h\|_{L_p(\Omega)} \quad \forall v_h \in V_h \quad (3.32)$$

for some given natural number p !