T U T O R I A L

"Numerical Methods for the Solution of Elliptic Partial Differential Equations"

to the lecture

"Numerics of Elliptic Problems"

Tutorial 06 Thursday, 14 April 2011, Time: $10^{\underline{15}}$ – $11^{\underline{45}}$, Room: SR / T 111.

3 Galerkin FEM

3.1 Galerkin-Ritz-Method

25 Let us consider the variational problem: Find $u \in V_g = V_0 = H_0^1(0,1)$:

$$\int_0^1 u'(x)v'(x)dx = \int_0^1 f(x)v(x)dx \quad \forall v \in V_0. \tag{3.13}$$

Solve this variational problem with the Galerkin-Method using the basis

$$V_{0h} = V_{0n} = \text{span}\{x(1-x), x^2(1-x), \ldots, x^{n-1}(1-x)\},$$

where the right-hand side is given as $f(x) = \cos(k\pi x)$, $k = l+1$ and l is the last digit from your study code (Matrikelnummer) ! Compute the stiffness matrix K_h analytically and solve the linear system $K_h \underline{u}_h = \underline{f}_h$ numerically using the Gaussian elimination method ! Consider n to be $2, 4, 8, 10, 50, 100$!

3.2 Net generation and refinement

26* In the lectures, we used the input file `*.net` (see Slide 10) for the input of the mesh data. Design and implement a new Algorithm, which inputs the file `coarse.net` containing a coarse triangulation and outputs the file `fine.net` containing the refinement of the coarse triangulation by dividing every triangle of the coarse mesh into 4 triangles (red refinement) !

27* How would you modify the algorithm from Exercise 26 in order to refine selected elements only ? Note that you have to ensure conformity of the triangulation by using the green refinement dividing a triangle into two triangles by bisection.

3.3 Mapping

28 Show the inequality

$$\frac{1}{2}\sin\theta_r\, h_r^2 \le |J_{\delta_r}| \le \frac{\sqrt{3}}{2} h_r^2, \tag{3.14}$$

where h_r is the longest side and θ_r the smallest angle of the triangle δ_r.