

■ Theorem 2.25: (Embedding for $W_p^1(\Omega)$)

Ass.: $\Omega \subset \mathbb{R}^d \neq \emptyset$ \wedge Lip, $1 < p, q < \infty$

St.: Statements 1.-3. remain valid if $\dot{W}_p^1(\Omega)$ is replaced by $W_p^1(\Omega)$.

■ Theorem 2.26: (Embedding for $W_p^k(\Omega)$)

Ass.: $\Omega \subset \mathbb{R}^d \neq \emptyset$ \wedge Lip, $1 < p, q < \infty$,

$0 \leq j < k$, $k = 1, 2, \dots$

St.: 1. $W_p^k(\Omega) \hookrightarrow C^j(\bar{\Omega})$ if $(k-j)p > d$.

2. Let $(k-j)p \leq d$.

Then the embeddings

$W_p^k(\Omega) \subset W_p^j(\Omega)$ resp. $\dot{W}_p^k(\Omega) \subset \dot{W}_p^j(\Omega)$
are

• continuous if $q \leq q_k := \frac{pd}{d-(k-j)p} \wedge q < \infty$.

• compact if $q < q_*$.

The results on the embedding of $\dot{W}_p^k(\Omega) \subset \dot{W}_q^j(\Omega)$ don't need the assumption that Ω is Lip.

■ Remarks:

1. St. 3 of Th. 2.24 and 2.25 can easily be generalized to $W_p^k(\Omega)$, i.e.

$W_p^k(\Omega) \subset W_q^j(\Omega_m)$ if (unns)

$W_p^k(\Omega) \hookrightarrow W_q^j(\Omega_m)$ if (unns)

2. All results are sharp!

For instance: $d=2$: $H^2(\Omega) \hookrightarrow C(\bar{\Omega})$, but
 $H^2(\Omega) \not\hookrightarrow C(\bar{\Omega})$!