As in the lecture, let $R_i^\top : V_i \to V$ be a linear and injective prolongation operator. Define the local bilinear form

$$a_i(v_i,\, w_i) \;:=\; a(R_i^\top v_i,\, R_i^\top w_i) \qquad \text{for } v_i,\, w_i \in V_i\,,$$

let $\widetilde{P}_i : V \to V_i$ be defined by

$$a_i(\widetilde{P}_i\, v,\, w_i) \;=\; a(v,\, R_i^\top w_i) \qquad \forall w_i \in V_i\,,$$

and let A, A_i be the operators associated to $a_i(\cdot,\cdot)$, $a(\cdot,\cdot)$, respectively.

04 Show that

(i) $A_i \;=\; R_i\, A\, R_i^\top$

(ii) $\widetilde{P}_i \;=\; A_i^{-1}\, R_i\, A$

05 Prove that $P_i := R_i^\top \widetilde{P}_i$ is a *projection*, i. e., $P_i^2 = P_i$.

Use the projection property to show that

$$a((I - P_i)v,\, P_i\, w) \;=\; 0.$$

06 Show that

$$a(P_{\mathrm{ad}}\, u,\, v) \;=\; \sum_{i=1}^{N} a_i(\widetilde{P}_i\, u,\, \widetilde{P}_i\, v) \;=\; a(u,\, P_{\mathrm{ad}}\, v),$$

where $P_{\mathrm{ad}} = \sum_{i=1}^{N} P_i$.

You may use all (proved) results from the lecture.