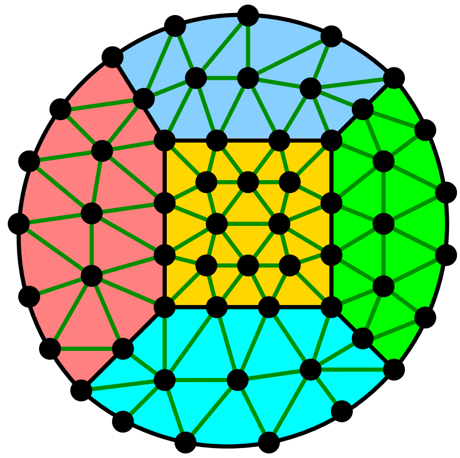
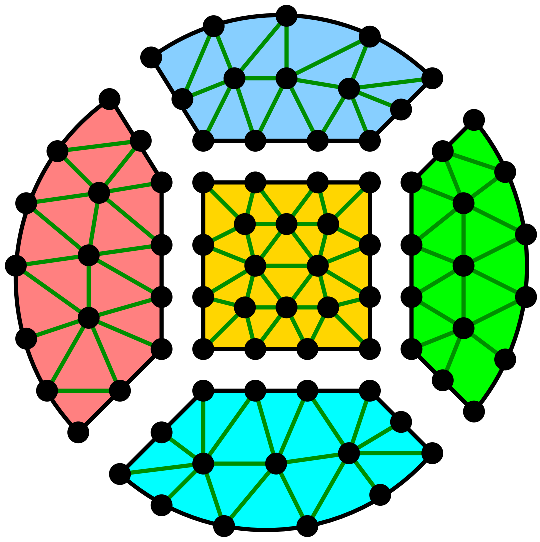
FETI – Idea

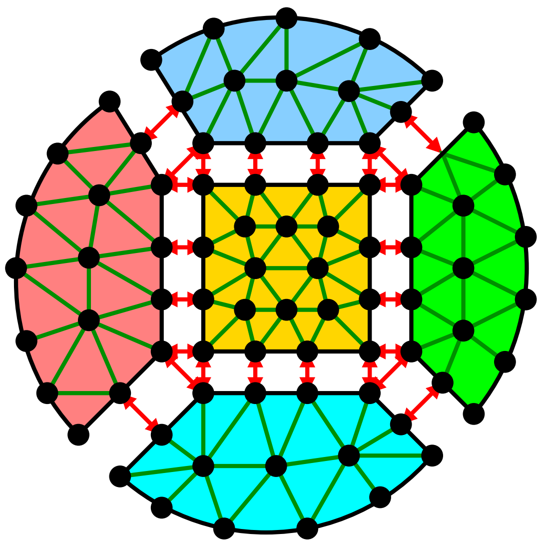
non-overlapping domain decomposition

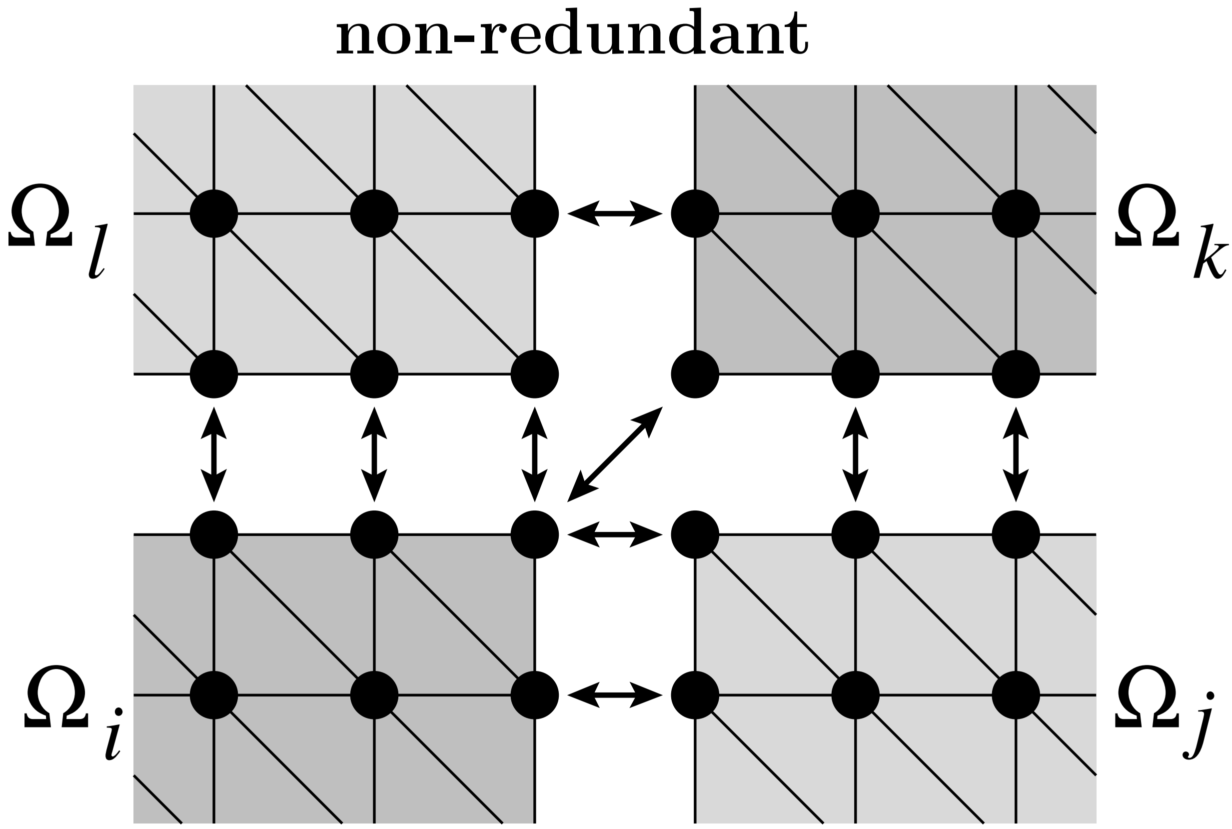


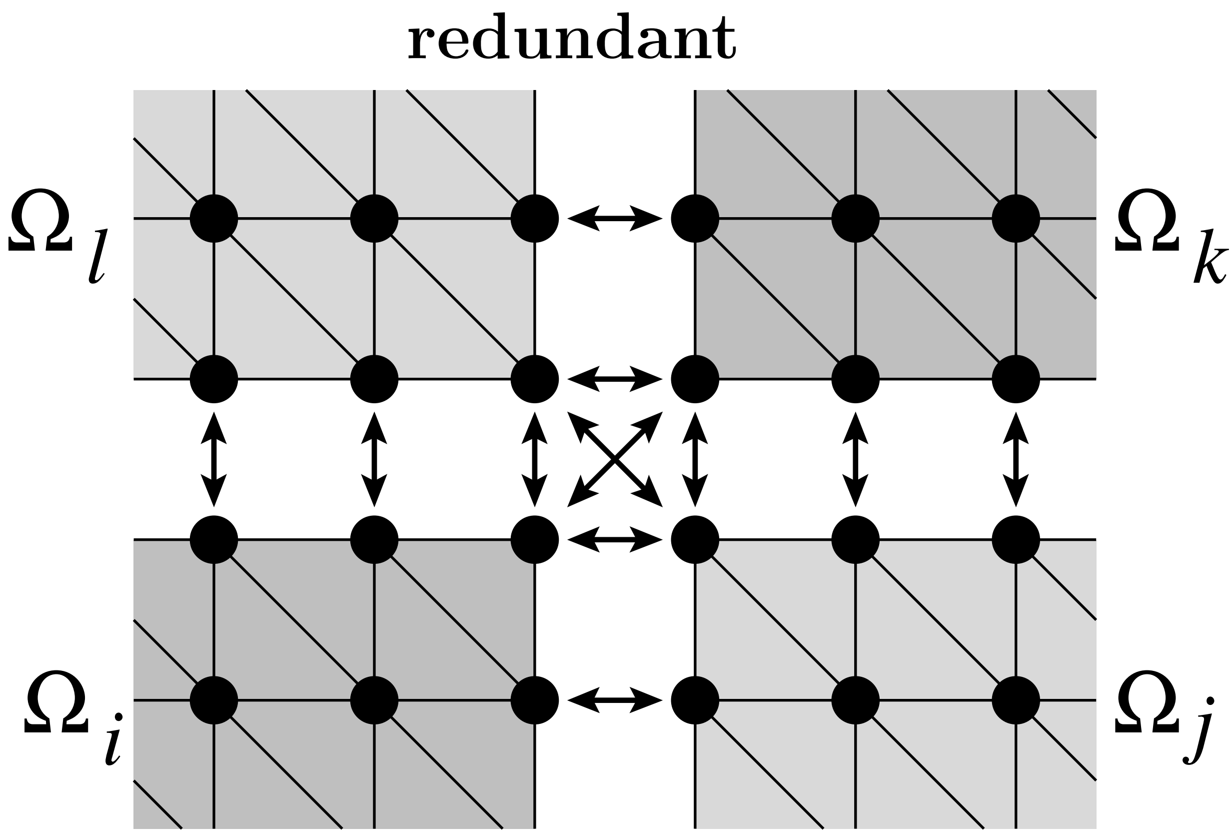
tearing



interconnecting




non-redundant
Ω_l
Ω_k
Ω_i
Ω_j


redundant
Ω_l
Ω_k
Ω_i
Ω_j

FETI – Derivation

Original problem equivalent to *constrained min. problem*. Using Lagrange multipliers:

$$\begin{bmatrix} A_1 & & & 0 & B_1^\top \\ & A_2 & & & B_2^\top \\ & & \ddots & & \vdots \\ 0 & & & A_N & B_N^\top \\ B_1 & B_2 & \cdots & B_N & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \\ \lambda \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_N \\ 0 \end{bmatrix}$$

If

$$R^T(f - B^T\lambda) = 0$$

then

$$u = A^\dagger(f - B^\top\lambda) + R\xi$$

Elimination of u:

$$\begin{bmatrix} F & -G \\ G^\top & 0 \end{bmatrix} \begin{bmatrix} \lambda \\ \xi \end{bmatrix} = \begin{bmatrix} BA^\dagger f \\ R^\top f \end{bmatrix}$$

$$(\; F = BA^\dagger B^\top \quad G = BR \;)$$

Projection method: $U_{\mathrm{ad}} := \{\lambda \in U : G^\top\lambda = 0\}$

$$\lambda = \underbrace{\lambda_0}_{:G^\top\lambda_0 = 0} + \underbrace{\widetilde{\lambda}}_{\in U_{\mathrm{ad}}}$$

$$P := I - QG(G^\top QG)^{-1}G^\top : U \to U_{\mathrm{ad}}$$
$$\lambda_0 = QG(G^\top QG)^{-1}R^\top f$$

Solve

$$P^\top F\widetilde{\lambda} = P^\top BA^\dagger(f - B^\top\lambda_0)$$

FETI – Overview

Spaces:

$$Y = \prod_{i=1}^{N} V_D^h(\Omega_i)$$ broken space of FE functions

$$U = \mathbb{R}^M$$ space of Lagrange multipliers

$$U_{\mathrm{ad}} = \{\lambda \in U : G^\top \lambda = 0\}$$

$$Z \cong \mathbb{R}^{N_{\mathrm{float}}}$$ parameter domain for $\ker(A)$

Operators:

$$A^\dagger : Y^* \longrightarrow Y \qquad B : Y \longrightarrow U^*$$

$$R : Z \longrightarrow Y \qquad f \in Y^*$$

More operators:

$$\begin{aligned} F &:= B A^\dagger B^\top &&: U \longrightarrow U^* \\ G &:= B R &&: Z \longrightarrow U^* \\ & Q &&: U^* \longrightarrow U \\ P &:= I - Q\,G(G^\top Q\,G)^{-1} G^\top &&: U \longrightarrow U_{\mathrm{ad}} \end{aligned}$$

Unpreconditioned FETI:

$\lambda_0 := Q\,G(G^\top Q\,G)^{-1} R^\top f$

$\widetilde{d} := P^\top B A^\dagger (f - B^\top \lambda_0)$

solve $P^\top F \widetilde{\lambda} = \widetilde{d}$ with CG and initial value 0

$\lambda := \lambda_0 + \widetilde{\lambda}$

$\xi := -(G^\top Q\,G)^{-1} G^\top Q B A^\dagger (f - B^\top \lambda)$

$u := A^\dagger (f - B^\top \lambda) + R\,\xi$