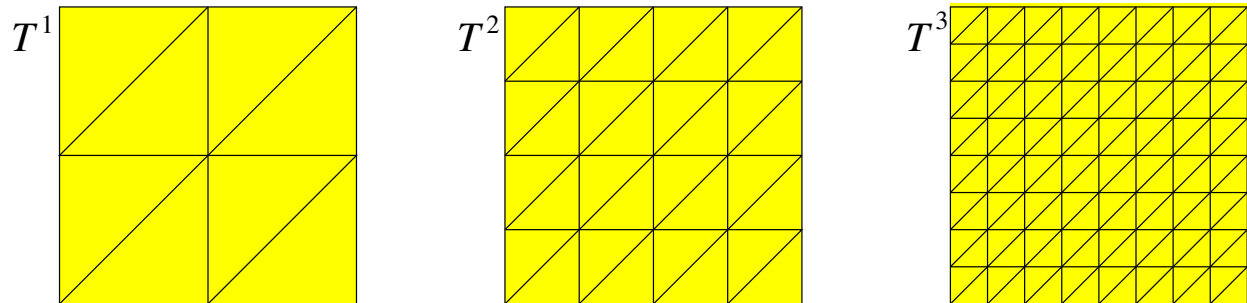


Multilevel Diagonal Scaling (MDS)



$$\mathcal{T}^\ell \leftrightarrow V^\ell \quad \ell = 1, \dots, L \quad V := V_L$$

$$V_{\ell,i} := \text{span}(\varphi_{\ell,u}) \quad A_{\ell,i} \leftrightarrow [\underline{A}_\ell]_{ii}$$

$$C_{\text{MDS}}^{-1} := \sum_{\ell=1}^L \sum_{i=1}^{n_\ell} R_{\ell,i}^\top A_{\ell,i}^{-1} R_{\ell,i}$$

Algorithm 1: Application of MDS

\underline{r}_L given

for $\ell = L, \dots, 2$ do

 | $\underline{r}_{\ell-1} = (\underline{I}_{\ell-1}^\ell)^\top \underline{r}_\ell$ (restrict)

end

for $\ell = 1, \dots, L$ do

 | $\underline{w}_\ell = \underline{C}_\ell^{-1} \underline{r}_\ell$ (apply classical Jacobi on level ℓ)

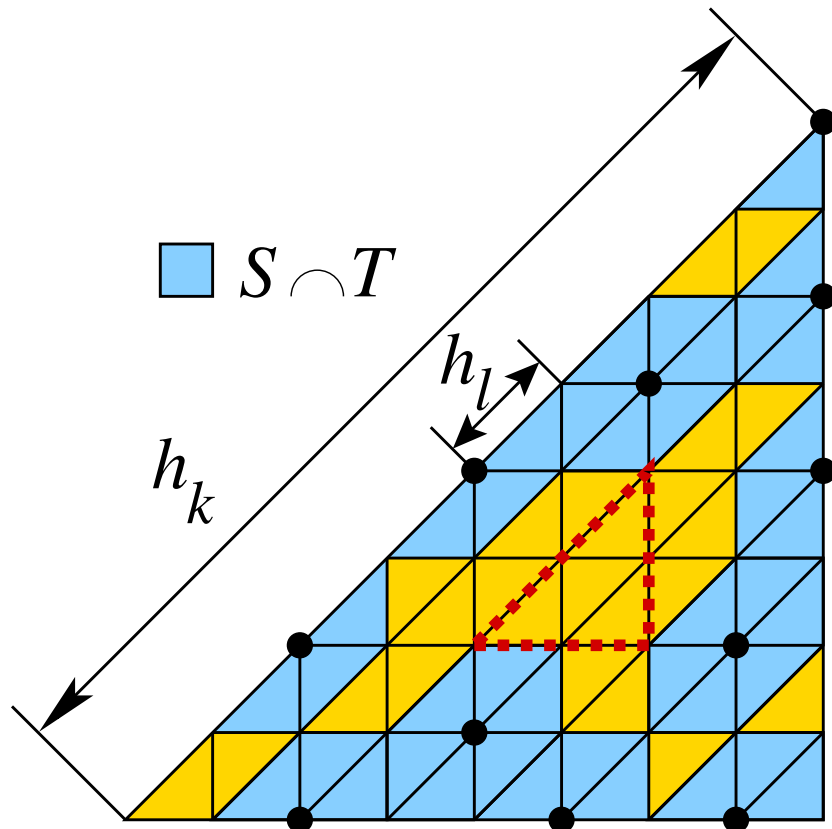
 if $i > 1$ then

 | $\underline{w}_\ell = \underline{w}_\ell + \underline{I}_{\ell-1}^\ell \underline{w}_{\ell-1}$ (add prolonged update)

 end

end

return \underline{w}_L



$P_\ell : V \rightarrow V_\ell$:

$$a(P_\ell v, w_\ell) = a(v, w_\ell) \quad \forall w_\ell \in V_\ell$$

Construction: $v \in V$:

$$v_k := \begin{cases} P_1 v & k = 1, \\ (P_k - P_{k-1})v & \text{else} \end{cases}$$

Splittings

$$P_\ell v = \sum_{k=1}^{\ell} v_k \quad v = \sum_{k=1}^L v_k$$

$$a(v, v) = \sum_{k=1}^L a(v_k, v_k)$$