# Two-level overlapping Schwarz

## Local solvers:

- overlapping partition $\Omega = \bigcup_{i=1}^{N} \Omega_i$
- associated subspaces $V_i \subset V$
  (functions vanish outside of $\Omega_i$)
- exact solvers on $V_i$

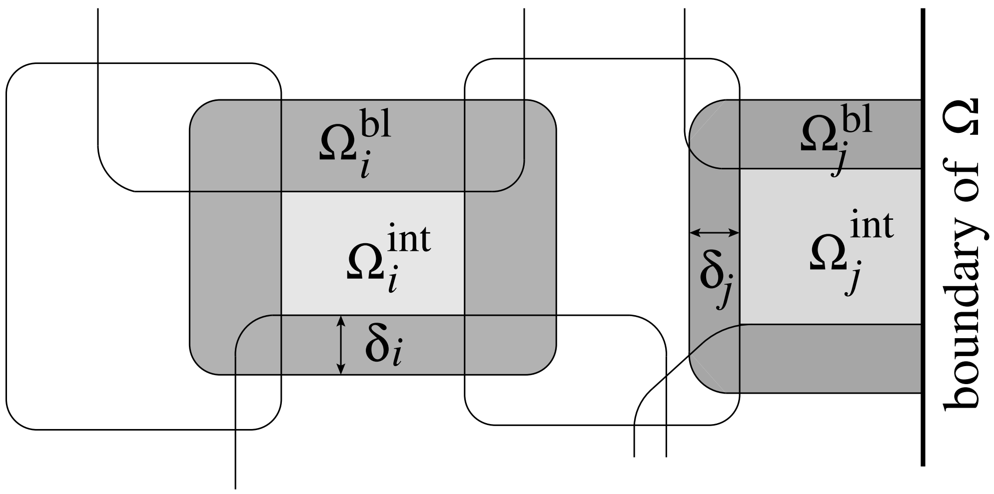
## Coarse solver:

- coarse mesh $\mathcal{T}^H(\Omega)$ (coarse and fine mesh nested)
- associated subspace $V_0 \subset V$
- exact solver on $V_0$

## Preconditioner:

$$C_{\mathrm{ad},2}^{-1} = \sum_{i=0}^{N} R_i^\top A_i^{-1} R_i$$

## Notation:




interior $\Omega_i^{\mathrm{int}}$
boundary layer $\Omega_i^{\mathrm{bl}}$
overlap parameter $\delta_i$

# Two-level overlapping Schwarz (analysis)

**Assumption A4.** $\forall i = 1, \ldots, N: \quad \delta_i > 0.$

**Assumption A5.** Each point $x \in \overline{\Omega}$ can only belong to at most $N_s$ of the subdomains $\{\Omega_i\}_{i=1}^N$.

**Assumption A6.** Coarse and fine mesh are *nested*.

**Assumption A7.** Each coarse element $T \in \mathcal{T}^H(\Omega)$ should not belong to more than $N_{cs}$ subdomains, and

$$\forall T \cap \Omega_i \neq \emptyset : \qquad H_T \lesssim \operatorname{diam}(\Omega_i).$$

## Coarse quasi-interpolator

$$\Pi^H : H_D^1(\Omega) \to V_D^H(\Omega)$$

$$(\Pi^H v)(x) = \begin{cases} 0 & x \in \Gamma_D \\ \overline{v}^{\omega_x} = \dfrac{1}{|\omega_x|} \displaystyle\int_{\omega_x} v(y)\, dy & \text{else} \end{cases}$$

for coarse node $x$.

# PARDISO
## factorization vs. solve time