4-16 • **Dann ZEITdiskretisierung mit Zeitintegrationsverfahren**

z.B. 1) Einschrittverfahren (expl. Euler, impl. Euler, CN, ...)
angewandt auf äquivalentes System gew. Dgl. 1. Ordn.:

$$(17)\quad \begin{cases} M_h \dot{\underline{v}}_h(t) = \underline{f}_h(t) - C_h(t)\underline{v}_h(t) - K_h(t)\underline{u}_h(t) \\ \dot{\underline{u}}_h(t) = \underline{v}_h(t) \\ + \text{AB}:\ \underline{u}_h(0) = M_h^{-1}\underline{u}_{0h},\ \underline{v}_h(0) = M_h^{-1}\underline{u}_{1h} \end{cases}$$

z.B. 2) Newmark-Verfahren [JL (2013), S. 567-569]
Btr. $(\underline{15})_h$ mit Dämpfungsterm

$$\underline{a}_h(t) \qquad \underline{v}_h(t)$$

$$(\underline{15})_h\quad \begin{cases} M_h \ddot{\underline{u}}_h(t) + C_h(t)\dot{\underline{u}}_h(t) + K_h(t)\underline{u}_h(t) = \underline{f}_h(t),\ t\in(0,T) \\ + \text{AB}:\ M_h\underline{u}_h(0) = \underline{u}_{0h},\ M_h\dot{\underline{u}}_h(0) = \underline{u}_{1h} \end{cases}$$

Newmark (β,γ): $\beta\in[0,1/2]$, $\gamma\in[0,1]$

Ges. $\underline{a}_h^{j+1}\in\mathbb{R}^{N_h}$ (Beschleunigung: $\underline{a}_h^{j+1}\approx\underline{a}_h(t_{j+1})$):

$$M_h\underline{a}_h^{j+1} + C_h(t_{j+1})\underline{v}_h^{j+1} + K_h(t_{j+1})\underline{u}_h^{j+1} = \underline{f}_h^{j+1} := \underline{f}_h(t_{j+1})$$

$$(18)\qquad \underline{u}_h^{j+1} = \underline{u}_h^j + \tau\underline{v}_h^j + \frac{\tau^2}{2}\left[(1-2\beta)\underline{a}_h^j + 2\beta\underline{a}_h^{j+1}\right]$$

Taylor: $\underline{u}_h(t_{j+1}) = \underline{u}_h(t_j) + \tau\frac{d\underline{u}_h}{dt}(t_j) + \frac{\tau^2}{2}\frac{d^2\underline{u}_h}{dt^2}(t_j+\theta_j\tau),\ \theta_j\in(0,1)$

$$\underline{v}_h^{j+1} = \underline{v}_h^j + \tau\left[(1-\gamma)\underline{a}_h^j + \gamma\underline{a}_h^{j+1}\right]$$

Taylor: $\underline{v}_h(t_{j+1}) = \underline{v}_h(t_j) + \tau\frac{d\underline{v}_h}{dt}(t_j+\tilde{\theta}_j\tau),\ \tilde{\theta}_j\in(0,1)$

$j = 0, 1, 2, \ldots, m-1$

$+$ AB: $\underline{u}_h^0 = M_h^{-1}\underline{u}_{0h}$, $\underline{v}_h^0 = M_h^{-1}\underline{u}_{1h}$

mit $\underline{a}_h^0 = M_h^{-1}\left[\underline{f}_h^0 - C_h(t_0)\underline{v}_h^0 + K_h(t_0)\underline{u}_h^0\right]$, $t_0 = 0$.

Daraus folgt sofort:

$$\underline{a}_h^{j+1}\in\mathbb{R}^{N_h}:\ \left(M_h + \tau\gamma C_h(t_{j+1}) + \tau^2\beta K_h(t_{j+1})\right)\underline{a}_h^{j+1} = \underline{\varphi}_h^{j+1},$$

mit $\underline{\varphi}_h^{j+1} = \underline{f}_h^{j+1} - C(t_{j+1})\underline{v}_h^j - \tau(1-\gamma)C_h(t_{j+1})\underline{a}_h^j -$
$- K_h(t_{j+1})\underline{u}_h^j - \tau K_h(t_{j+1})\underline{v}_h^j - \frac{\tau^2}{2}(1-2\beta)K_h(t_{j+1})\underline{a}_h^j$

Bem.: Newmark $(\beta=\frac{1}{4},\gamma=\frac{1}{2})$ = CN = TR angew. auf (17)! unbed. stabil