Galerkin-Ritz-FEM

Variationsformulierung	Minimumproblem
$u \in \tilde{V}_g : a(u,v) = \langle F, v \rangle \;\; \forall v \in \tilde{V}_0$ $\Longleftrightarrow$	$u \in \tilde{V}_g : J(u) = \min_{v \in \tilde{V}_g} J(v)$

$a(\cdot,\cdot)$ sym., positiv

$\tilde{V}_{gh} \subset \tilde{V}_g$

$\tilde{V}_{0h} \subset \tilde{V}_0$

$V_{gh} \subset \tilde{V}_g$

Galerkin	Ritz
$u_h \in \tilde{V}_{gh} : a(u_h, v_h) = \langle F, v_h \rangle \;\; \forall v_h \in \tilde{V}_{0h}$ $\Longleftrightarrow$	$u_h \in \tilde{V}_{gh} : J(u_h) \le J(v_h) \;\; \forall v_h \in \tilde{V}_{gh}$

$v_h = \varphi^{(j)}$, $j \in \omega_h$

$$u_h = \sum_{i \in \omega_h} u^{(i)} \varphi^{(i)} + \sum_{i \in \gamma_h} u_*^{(i)} \varphi^{(i)}$$

$\dfrac{\partial J(\underline{u}_h)}{\partial u^{(j)}} = 0$, $j \in \omega_h$

Galerkin-Ritz-System = endlichdim. GS

Ges. $\underline{u}_h = [u^{(i)}]_{i \in \omega_h} \in \mathbb{R}^{N_h}$:

$$\sum_{i \in \omega_h} u^{(i)} a(\varphi^{(i)}, \varphi^{(j)}) = \langle F, \varphi^{(j)} \rangle - \sum_{i \in \gamma_h} u_*^{(i)} a(\varphi^{(i)}, \varphi^{(j)}) \qquad j \in \omega_h$$

$$K_h \underline{u}_h = \underline{f}_h$$

mit $J(v) := \frac{1}{2} a(v,v) - \langle F, v \rangle$ – Ritzsches Energiefunkt.