Hauptsatz der Integral- und Differentialrechnung

$d=1$ | $d=1,2,3$

(3) $\int_a^b w'(x)\,dx = w(b)-w(a)$ $\qquad$ $\int_\Omega \frac{\partial w}{\partial x_i}(x)\,dx = \int_{\partial\Omega} w(x)n_i(x)\,ds$

$$\Big\downarrow\; w = u\cdot\varphi$$

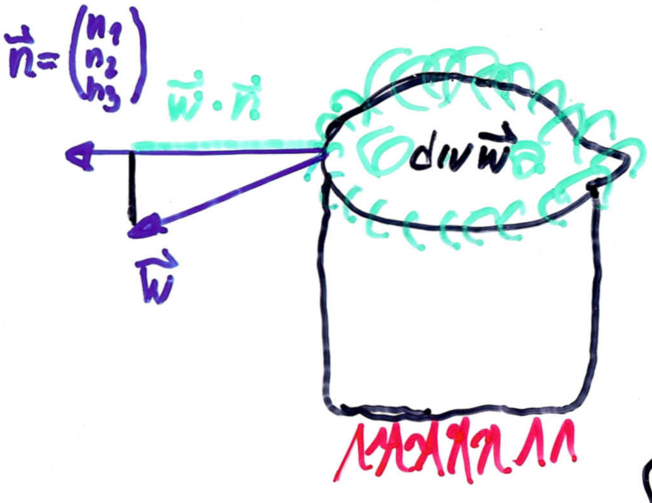
(2) $\int_a^b u\varphi'\,dx = -\int_a^b u'\varphi\,dx + \underbrace{u\varphi\Big|_a^b}_{=0}$ $\qquad$ $\int_\Omega u\frac{\partial\varphi}{\partial x_i}\,dx = -\int_\Omega \frac{\partial u}{\partial x_i}\varphi\,dx + \underbrace{\int_{\partial\Omega} u\varphi n_i\,ds}_{=0}$

$\varphi(a)=\varphi(b)=0$ $\qquad$ $\varphi=0$ auf $\partial\Omega$

Formel der partiellen Integration

__Übung:__ Zeigen Sie mit Hilfe von (3) den Gaußschen Integralsatz

(4) $\int_\Omega \operatorname{div}\vec{w}\,dx = \int_\Omega \vec{w}\cdot\vec{n}\,ds \quad \forall \vec{w} = \begin{pmatrix} w_1\\ w_2\\ w_3\end{pmatrix} \in C^1(\bar{\Omega})$



__Beweis:__ trivial

$w = w_i$ in (3)

$\sum_{i=1}^d$

$$\int_\Omega \underbrace{\sum_{i=1}^d \frac{\partial w_i}{\partial x_i}}_{\operatorname{div}\vec{w}}\,dx = \int_{\partial\Omega} \underbrace{\sum_{i=1}^d w_i n_i}_{\vec{w}\cdot\vec{n}}\,ds$$