Preconditioned Conjugate Gradient (PCG) Method:

- **PCG - Algrithms:**

Startschritt:

$x^{(0)} \in \mathbb{R}^n$ - geg. Startnähr., z.B. $x^{(0)} = C^{-1}b$

$d^{(0)} := b - Ax^{(0)}$

$w^{(0)} := C^{-1} * d^{(0)}$

$s^{(0)} := w^{(0)}$

Iteration: $k = 0, 1, \ldots$ (Konvergenztest)

Test: $(w^{(k)}, d^{(k)}) \leq \varepsilon^2 (w^{(0)}, d^{(0)})$

$\| z^{(k)} \|_{AC^{-1}A} \leq \varepsilon \| z^{(0)} \|_{AC^{-1}A}$ (mms)

$$\alpha^{(k+1)} := \frac{(d^{(k)}, s^{(k)})}{(As^{(k)}, s^{(k)})} \underset{(mms)}{=} \frac{(d^{(k)}, w^{(k)})}{(As^{(k)}, s^{(k)})}$$

$x^{(k+1)} := x^{(k)} + \alpha^{(k+1)} s^{(k)}$

$d^{(k+1)} := d^{(k)} - \alpha^{(k+1)} A s^{(k)}$

$w^{(k+1)} := C^{-1} \times d^{(k+1)}$

$$\beta^{(k+1)} := \frac{(Aw^{(k+1)}, s^{(k)})}{(As^{(k)}, s^{(k)})} \overset{(mms)}{=} \frac{(w^{(k+1)}, d^{(k+1)})}{(w^{(k)}, d^{(k)})}$$

$s^{(k+1)} := w^{(k+1)} + \beta^{(k+1)} s^{(k)} \perp s^{(k)}$