2. Elementsteifigkeitmatrizen $K^{(r)} = [K^{(r)}_{\alpha\beta}]_{\alpha,\beta=1}^{3}$:

$$K^{(r)}_{\alpha\beta} = \int_{\delta_r} \Big[\lambda_1(x) \frac{\partial \varphi^{(r)}_\beta(x)}{\partial x_1} \frac{\partial \varphi^{(r)}_\alpha(x)}{\partial x_1} + \lambda_2(x) \frac{\partial \varphi^{(r)}_\beta(x)}{\partial x_2} \frac{\partial \varphi^{(r)}_\alpha(x)}{\partial x_2} +$$

$$+ a(x)\, \varphi^{(r)}_\beta(x)\, \varphi^{(r)}_\alpha(x) \Big]\, dx$$

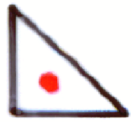
$\delta_r \longrightarrow \Delta \qquad \dfrac{\partial}{\partial x_1} = \dfrac{\partial}{\partial \xi_1}\dfrac{\partial \xi_1}{\partial x_1} + \dfrac{\partial}{\partial \xi_2}\dfrac{\partial \xi_2}{\partial x_1} = \ldots$

$$= \int_\Delta \Big[\lambda_1(x_{\delta_r}(\xi)) \Big(\frac{\partial \phi_\beta}{\partial \xi_1}\frac{\partial \xi_1}{\partial x_1} + \frac{\partial \phi_\beta}{\partial \xi_2}\frac{\partial \xi_2}{\partial x_1} \Big) \Big(\frac{\partial \phi_\alpha}{\partial \xi_1}\frac{\partial \xi_1}{\partial x_1} + \frac{\partial \phi_\alpha}{\partial \xi_2}\frac{\partial \xi_2}{\partial x_1} \Big) +$$

$$+ \lambda_2(x_{\delta_r}(\xi)) \Big(\frac{\partial \phi_\beta}{\partial \xi_1}\frac{\partial \xi_1}{\partial x_2} + \frac{\partial \phi_\beta}{\partial \xi_2}\frac{\partial \xi_2}{\partial x_2} \Big) \Big(\frac{\partial \phi_\alpha}{\partial \xi_1}\frac{\partial \xi_1}{\partial x_2} + \frac{\partial \phi_\alpha}{\partial \xi_2}\frac{\partial \xi_2}{\partial x_2} \Big) +$$

$$+ a(x_{\delta_r}(\xi))\, \phi_\beta(\xi)\, \phi_\alpha(\xi) \Big]\, |J_{\delta_r}|\, d\xi$$

$$\approx [\quad -\text{''}- \quad]_{\xi=\xi_*}\, |J_{\delta_r}|_{\xi=\xi_*} \underbrace{\text{meas}\,\Delta}_{=1/2}$$



$\xi_* = \left(\frac{1}{3}, \frac{1}{3}\right)$

mit (im Falle linearer Formfunktionen)

$\phi_1(\xi) = 1 - \xi_1 - \xi_2 \qquad \dfrac{\partial \phi_1}{\partial \xi_1} = -1 \qquad \dfrac{\partial \phi_1}{\partial \xi_2} = -1$

$\phi_2(\xi) = \xi_1 \qquad \dfrac{\partial \phi_2}{\partial \xi_1} = 1 \qquad \dfrac{\partial \phi_2}{\partial \xi_2} = 0$

$\phi_3(\xi) = \xi_2 \qquad \dfrac{\partial \phi_3}{\partial \xi_1} = 0 \qquad \dfrac{\partial \phi_3}{\partial \xi_2} = 1$

$$\boxed{K^{(r)}_{\alpha\beta} := [-\text{''}-]_{\xi=\xi_*}\, |J_{\delta_r}| \cdot \frac{1}{2}}$$

Ausgeschriebene Formeln siehe Skriptum, Abs. 4.4.3