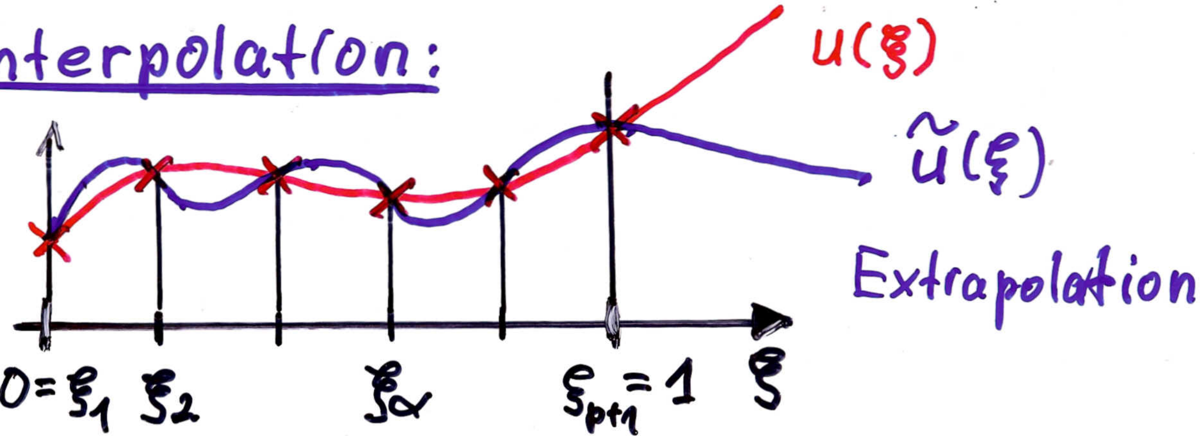
Anwendungen der Lagrangen Interpolationspolynome:

a) Interpolation:



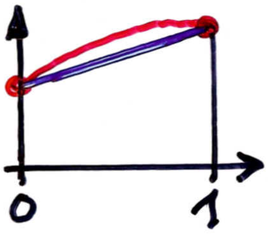
$$\tilde{u}(\xi) := \sum_{\alpha=1}^{p+1} \overset{\times}{\underset{\underset{u(\xi_\alpha)}{\shortparallel}}{u_\alpha}} \, \Phi_\alpha(\xi) \approx u(\xi)$$

b) Numerische Integration: Newton-Cotes-Formeln

$$\int_0^1 u(\xi)\,d\xi \approx \int_0^1 \tilde{u}(\xi)\,d\xi = \sum_{\alpha=1}^{p+1} u(\xi_\alpha) \underbrace{\int_0^1 \Phi_\alpha(\xi)\,d\xi}_{=:\, w_\alpha}$$

$$= \sum_{\alpha=1}^{p+1} \underset{\text{Stützstellen}}{\underset{\uparrow}{u(\xi_\alpha)}} \; \underset{\text{Gewichte}}{\underset{\uparrow}{w_\alpha}}$$

Bsp.: $p=1$

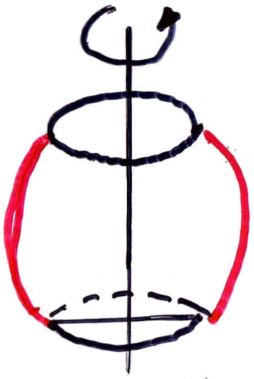


Trapezregel

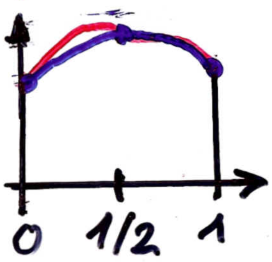
$\xi_1=0$: $w_1 = \int_0^1 (1-\xi)\,d\xi = 1/2$

$\xi_2=1$: $w_2 = \int_0^1 \xi\,d\xi = 1/2$

$\int_0^1 u(\xi)\,d\xi \approx \frac{1}{2}(u(0)+u(1))$



$p=2$



Keplersche Faßregel

$\xi_1=0$: $w_1 = \int_0^1 (2\xi^2-2\xi+1)\,d\xi = \frac{1}{6}$

$\xi_2=1/2$: $w_2 = \int_0^1 (-4\xi^2+4\xi)\,d\xi = \frac{4}{6}$

$\xi_3=1$: $w_3 = \int_0^1 (2\xi^2-\xi)\,d\xi = 1/6$

$\int_0^1 u(\xi)\,d\xi \approx \frac{1}{6}(u(0)+4u(\frac{1}{2})+u(1))$