T U T O R I A L

"Computational Mechanics"

to the lecture

"Numerical Methods in Continuum Mechanics 1"

Tutorial 13/14 Wednesday, Jun 23, 2010
(Time : $8^{\underline{30}}$ – $10^{\underline{00}}$ Room : SR T 111)

0.1 The Hellinger-Reisner Principle

44 Let $\bar{u} \in H^{1/2}(\Gamma_u)$ and $\sigma_t \in H(\mathrm{div}, \Omega)$ be given vector and tensor functions. Show that the functional F defined by the identity

$$\langle F \, , \, \tau \rangle := \left(D^{-1} \sigma_t \, , \, \tau \right)_0 + \int_{\Gamma_u} (\tau n) \cdot \bar{u} \, \mathrm{d}s \, ,$$

belongs to X_0^*, where $X_0 := \{ \sigma \in H(\mathrm{div}, \Omega) \mid \sigma n = 0 \text{ on } \Gamma_t \}$.

45 Let $v \in L_2(\Omega)$ be a given vector function. Let $u \in H^1_{0,\Gamma_u}(\Omega)$ be such that

$$(\varepsilon(u) \, , \, \varepsilon(w))_0 = - (v \, , \, w)_0 \; , \quad \forall w \in H^1_{0,\Gamma_u}(\Omega) \, .$$

Show that $\tau := \varepsilon(u)$ is in $X = H(\mathrm{div}, \Omega)$, and that $\mathrm{div}\, \tau = v$.

46* Consider the definitions in Example **45**. Show that $\tau_n (= \tau n) = 0$ on Γ_t in the sense

$$\langle \tau n \, , \, w \rangle_{H^{-1/2}(\Gamma_t) \times H^{1/2}(\Gamma_t)} = 0 \, , \quad \forall w \in H^1_{0,\Gamma_u}(\Omega) \, .$$