T U T O R I A L

"Computational Mechanics"

to the lecture

"Numerical Methods in Continuum Mechanics 1"

Tutorial 09 Friday, May 28, 2010 (Time : $10^{\underline{15}}$ – $11^{\underline{00}}$, Room : HS 14)

4.3 Solvers for Mixed Finite Element Equations

32 Let $A \in R^{n\times n}$, $B \in R^{m\times n}$, and $C \in R^{m\times m}$ such, that $A = A^T$, $A > 0$, $C = C^T$, $C \geq 0$, and $\operatorname{Rank} B = \min\{m, n\}$. Show that

1. the matrix $\begin{pmatrix} A & B^T \\ B & -C \end{pmatrix}$ is symmetric but indefinite, and
2. if $C > 0$, then the matrix $\begin{pmatrix} A & B^T \\ -B & C \end{pmatrix}$ is positive definite !

33 Let $A \in R^{n\times n}$, $B \in R^{m\times n}$, and $C \in R^{m\times m}$ such, that $A = A^T$, $A > 0$, $C = C^T$, $C \geq 0$, and $\operatorname{Rank} B = \min\{m, n\}$. Show that the dual Schur complement matrix $S = BA^{-1}B^T + C$ is symmetric and positive definite (SPD) !

34* Show that the best (largest) discrete LBB-constant is given by the relation

$$\tilde{\beta}_1^2 = \lambda_{min}(G_\Lambda^{-1}(BG_X^{-1}B^T)), \tag{4.60}$$

i.e. $\tilde{\beta}_1^2 = \lambda_{min}(G_\Lambda^{-1}(BG_X^{-1}B^T))$ is the minimal eigenvalue of the generalized EVP

$$BG_X^{-1}B^T\underline{\mu} = \lambda G_\Lambda \underline{\mu}, \tag{4.61}$$

where the Gram matrices G_X and G_Λ are defined by the identities

$$(G_X\underline{u}_h, \underline{v}_h)_{\mathbf{R}^{n_h}} = (u_h, v_h)_X \quad \forall \underline{u}_h, \underline{v}_h \leftrightarrow u_h, v_h \in X_h \quad \text{and}$$

$$(G_\Lambda\underline{\lambda}_h, \underline{\mu}_h)_{\mathbf{R}^{m_h}} = (\lambda_h, \mu_h)_\Lambda \quad \forall \underline{\lambda}_h, \underline{\mu}_h \leftrightarrow \lambda_h, \mu_h \in \Lambda_h, \quad \text{respectively.}$$

Hint: Use the Rayleigh-quotient representation of the minimal eigenvalue of the generalized EVP (4.61) !

35 Write down the Uzawa–CG–Method, i.e., the CG–Method for the Schur-Complement-System

$$\text{Given } \underline{f} \in \mathbf{R}^n \text{ and } \underline{g} \in \mathbf{R}^m \text{. Find } \underline{\lambda} \in \mathbf{R}^m : \quad \left(BA^{-1}B^T + C\right)\underline{\lambda} = BA^{-1}\underline{f} - \underline{g}$$

in an algorithmic form !