TUTORIAL

"Computational Mechanics"

to the lecture

"Numerical Methods in Continuum Mechanics 1"

Tutorial 07 Friday, May 14, 2010 (Time : $10^{\underline{15}} - 11^{\underline{00}}$, Room : HS 14)

25* Consider the mixed variational problem: Find $(u, \lambda) \in X \times \Lambda$ such that

$$\begin{aligned} a(u,v) + b(v,\lambda) &= \langle F\,,\,v\rangle \quad \forall v \in X\,,\\ b(u,\mu) &= \langle G\,,\,\mu\rangle \quad \forall \mu \in \Lambda\,, \end{aligned}$$

where $F \in X^*$ and $G \in \Lambda^*$ are given. Let $A : X \to X^*$ and $B : X \to \Lambda^*$ be the related operators to $a(\cdot,\cdot)$ and $b(\cdot,\cdot)$, and let the assumptions of Theorem 2.4 (*Brezzi*) be satisfied. Show that the bilinearform

$$l(\xi,\eta) := \langle L\,\xi\,,\,\eta\rangle$$

with

$$L := \begin{pmatrix} A & B^* \\ B & 0 \end{pmatrix}, \ \xi := \begin{pmatrix} u \\ \lambda \end{pmatrix}, \ \eta := \begin{pmatrix} v \\ \mu \end{pmatrix}, \text{ and } \|\xi\|_{X\times\Lambda} = \left(\|u\|_X^2 + \|\lambda\|_\Lambda^2\right)^{1/2}$$

satisfies the assumptions of Theorem 1.5 (*Babuska-Aziz*), if $a(\cdot,\cdot)$ is elliptic on the whole space X, i. e., if there exists $\alpha_1 > 0$ such that $a(v,v) \geq \alpha_1\|v\|_X^2$ for all $v \in X$.

Hint: The LBB-condition

$$\exists \mu_1 > 0\ \forall \xi = (u,\lambda) \in X \times \Lambda :\ \sup_\eta \frac{l(\xi,\eta)}{\|\eta\|} \geq \mu_1 \|\xi\|$$

can be shown by choosing $\eta = (v,\mu)$ such that $\mu = -2\lambda$, and $v = u + w$ where $w \in X$ is the solution of the adjoint problem $a(y,w) = b(y,\lambda)$ for all $y \in X$.

26 Let the assumptions of Theorem 2.7 be fulfilled. Additionally we assume $\operatorname{Ker} B_h \subset \operatorname{Ker} B$, and define $Z_h(G) = \{v_h \in X_h \mid b(v_h,\mu_h) = \langle G\,,\,\mu_h\rangle\ \ \forall \mu_h \in \Lambda_h\}$. Show that there holds the estimate

$$\|u - u_h\|_X \leq \left(1 + \frac{\alpha_2}{\tilde{\alpha}_1}\right) \inf_{v_h \in Z_h(G)} \|u - v_h\|_X\,. \tag{4.24}$$

27 Let the operators $B^* : \Lambda \to X^*$ and $B_h^* : \Lambda_h \to X_h^*$ be defined by

$$\langle B^*\mu\,,\,v\rangle = b(v,\mu) \quad \forall v \in X\ \forall \mu \in \Lambda$$

and

$$\langle B_h^*\mu_h\,,\,v_h\rangle = b(v_h,\mu_h) \quad \forall v_h \in X_h\ \forall \mu_h \in \Lambda_h\,.$$

Show that, if there exists a linear operator $\Pi_h : X \to X_h$ with

$$b(\Pi_h v - v,\mu_h) = 0 \quad \forall v \in X\ \forall \mu_h \in \Lambda_h\,,$$

then there holds $\operatorname{Ker} B_h^* \subset \operatorname{Ker} B^*$.