T U T O R I A L

"Computational Mechanics"

to the lecture

"Numerical Methods in Continuum Mechanics 1"

Tutorial 06 Friday, May 7, 2010 (Time : $10^{\underline{15}}$ – $11^{\underline{00}}$, Room : HS 14)

22 Show directly (without using Theorem 2.4 (*Brezzi*)), that under the assumptions of Theorem 2.4 (*Brezzi*) the homogeneous mixed variational problem

$$\begin{aligned} a(u,v) + b(v,\lambda) &= 0 \quad \forall\, v \in X \\ b(u,\mu) &= 0 \quad \forall\, \mu \in \Lambda \end{aligned}$$

has only the trivial solution $(u,\lambda) = (0,0) \in X \times \Lambda$!

4.2 Mixed Finite Element Methods

23 Consider the problem: Find $u \in U$ such that for given $F \in V^*$ there holds

$$a(u,v) = \langle F\,,\, v\rangle \quad \forall v \in V\,.$$

Let the assumptions of Theorem 1.5 (*Babuska-Aziz*) be satisfied, and let $U_h \subset U$ and $V_h \subset V$ be finite dimensional subspaces. Further, we assume

$$\exists \tilde{\mu}_1 > 0 : \qquad \inf_{\substack{u_h \in U_h \\ u_h \neq 0}} \sup_{\substack{v_h \in V_h \\ v_h \neq 0}} \frac{a(u_h, v_h)}{\|u_h\|_U \|v_h\|_V} \geq \tilde{\mu}_1\,, \tag{4.20}$$

and

$$\forall v_h \in V_h,\ v_h \neq 0\ \exists u_h \in U_h : \quad a(u_h, v_h) \neq 0\,. \tag{4.21}$$

Show, that there exists a unique solution to the variational problem:

$$\text{Find } u_h \in U_h : \quad a(u_h, v_h) = \langle F\,,\, v_h\rangle \quad \forall v_h \in V_h\,, \tag{4.22}$$

24 Also show for problem described in Exercise 23 that the discretization error can be estimated from above by

$$\|u - u_h\|_U \leq \left(1 + \frac{\mu_2}{\tilde{\mu}_1}\right) \inf_{w_h \in U_h} \|u - w_h\|_U\,. \tag{4.23}$$