# T U T O R I A L

## "Computational Mechanics"

to the lecture

"Numerical Methods in Continuum Mechanics 1"

**Tutorial 04** Friday, April 16, 2010 (Time : $10^{\underline{15}}$ – $11^{\underline{00}}$, Room : HS 14)

## 3 Nonlinear Variational Problems and Variational Inequalities

**14** Let us consider the abstract nonlinear variational problem (15) from Transparency 04 under the assumption made there. Show that there exists a unique solution $u \in V_0$ of the nonlinear variational problem (15) and that the fixed point iteration (17) converges to this solution !

**15** Let us consider the abstract nonlinear variational problem (15) from Transparency 04 under the assumption made there, and its finite element approximation: find $u_h \in V_{0h} \subset V_0$ such that

$$a(u_h, v_h) = \langle f, v_h \rangle \quad \forall v_h \in V_{0h}. \tag{3.13}$$

Show the Cea-like discretization error estimate

$$\|u - u_h\|_{V_0} \leq \frac{\mu_2}{\mu_1} \inf_{w_h \in V_{0h}} \|u - w_h\|_{V_0}, \tag{3.14}$$

where the $\mu_1$ and $\mu_2$ are the monotonicity and the Lipschitz constants, respectively.

**16** Show that the variational inequality (19) is equivalent to the minimization problem (20) if the bilinear form is additionally symmetric (see Transparency 05) !

**17*** Show that if $f$ and $g$ are additionally continuous on $\bar{\Omega}$, a solution $u \in U \cap C^2(\bar{\Omega})$ of the obstacle problem (MP) $\equiv$ (VI) satisfies the PDE (in)equilities $-\Delta u \geq f$, $u \geq g$, $(\Delta u + f)(u - g) = 0$ in $\Omega$ and $u = 0$ on $\Gamma$ (see Transparency 06) !