T U T O R I A L

"Computational Electromagnetics"

to the lecture

"Numerical Methods in Electrical Engineering"

Tutorial 06 Thursday, June 10, 2010 (Time: 15:30 – 16:15; Room: T 212)

Beside the magnetostatic variational problem (problem (16) in Chapter 2 of the Lectures)

$$\text{Find } u \in V_0 = H_0(curl) : \; a(u,v) = \langle F, v\rangle \quad \forall v \in V_0, \tag{2.1}$$

we consider the regularized magnetostatic variational problem

$$\text{Find } u_\varepsilon \in V_0 = H_0(curl) : \; a_\varepsilon(u_\varepsilon,v) = \langle F, v\rangle \quad \forall v \in V_0, \tag{2.2}$$

where $\varepsilon \in \mathbf{R}^+$ is some positive regularization parameter, and the linear form and the bilinear forms are given by the following identities:

$$\langle F, v\rangle \;:=\; \int_\Omega f(x)\cdot v(x)\,dx \quad \forall v \in V_0, \tag{2.3}$$

$$a_\varepsilon(u,v) \;:=\; \int_\Omega (\nu(x)\,\mathrm{curl}(u(x))\cdot \mathrm{curl}(v(x)) + \varepsilon\, u(x)\cdot v(x))\,dx \quad \forall u,v \in V_0, \tag{2.4}$$

$$a(u,v) \;:=\; a_0(u,v) \;:=\; \int_\Omega \nu(x)\,\mathrm{curl}(u(x))\cdot \mathrm{curl}(v(x))\,dx \quad \forall u,v \in V_0. \tag{2.5}$$

We assume that the assumptions (A1) $\nu \in L_\infty(\Omega) : 0 < \nu_1 \leq \nu(x) \leq \nu_2$ for almost all $x \in \Omega$ and (A3) $f \in H(div, 0)$, i.e. $f \in (L_2(\Omega))^3$ with $\mathrm{div}(f) = 0$ are fulfilled.

20 Show that the regularized magnetostatic problem (2.2) has a unique solution $u_\varepsilon \in V_0$ satisfying the a priori estimate $\|u_\varepsilon\|_{L_2(\Omega)} \leq \varepsilon^{-1}\|f\|_{L_2(\Omega)}$.

21 Let $u \in V_0$ be that solution of the original magnetostatic problem which is L_2-orthogonal to the gradients of H^1_0-functions, i.e. $(u, \nabla\psi)_{L_2(\Omega)} = 0$ for all $\psi \in H^1_0(\Omega)$. Show that

$$\|u - u_\varepsilon\|_{H(curl)} \leq c\,\|f\|_{L_2(\Omega)}, \tag{2.6}$$

where $u_\varepsilon \in V_0$ is the solution of the regularized magnetostatic problem (2.2) and c is some positive constant.

In addition to the primale variational formulation of the regularized magnetostatic problem (2.2) we now consider the mixed variational problem: Find $(u_\varepsilon, \varphi_\varepsilon) \in V_0 \times \Lambda := H_0(curl,\Omega) \times H^1_0(\Omega)$ such that

$$\int_\Omega (\nu\,\mathrm{curl}(u_\varepsilon)\cdot \mathrm{curl}(v) + \varepsilon\, u_\varepsilon \cdot v)\,dx + \int_\Omega v\cdot\nabla\varphi_\varepsilon\,dx \;=\; \int_\Omega f\cdot v\,dx \quad \forall v \in V_0, \tag{2.7}$$

$$\int_\Omega u_\varepsilon\cdot\nabla\psi\,dx \;=\; 0 \quad \forall \psi \in \Lambda \tag{2.8}$$

22 Show that the regularized magnetostatic mixed variational problem (2.7)-(2.8) has a unique solution $(u_\varepsilon, \varphi_\varepsilon) \in V_0 \times \Lambda$, and, for u_ε, the a priori estimate

$$\|u_\varepsilon\|_{H(curl)} \leq c\,\|f\|_{L_2(\Omega)} \tag{2.9}$$

is valid.

Hint: Use Brezzi's Theorem B4 !

23 Show the following statement: If $(u_\varepsilon, \varphi_\varepsilon) \in V_0 \times \Lambda$ denotes the unique solution of the regularized magnetostatic mixed variational problem (2.7)-(2.8), then $u_\varepsilon \in V_0$ solves the primale regularized magnetostatic variational problem (2.2) !

24* Let $(u = u_0, \varphi = 0) \in V_0 \times \Lambda$ be the solution of the original mixed variational problem (= problem (19) in Chapter 2 of the Lectures = (2.7)-(2.8) for $\varepsilon = 0$)). Show that then the error estimate

$$\|u - u_\varepsilon\|_{H(curl)} \leq c\,\varepsilon\,\|f\|_{L_2(\Omega)} \tag{2.10}$$

is valid, where $u_\varepsilon \in V_0$ is the primal part of the solution of the regularized magnetostatic mixed variational problem (2.7)-(2.8).

Hint: Use Brezzi's Theorem B4 !