T U T O R I A L

"Computational Electromagnetics"

to the lecture

"Numerical Methods in Electrical Engineering"

Tutorial 05 Thursday, May 20, 2010 (Time: 15:30 – 16:15; Room: T 212)

16 Let $q \in [L_2(\Omega)]^3$ be a given vector function, and let us consider the BVP: Find $\varphi \in H^1(\Omega)$ such that

$$(\nabla\varphi, \nabla v)_{L_2(\Omega)} + \int_\Omega \varphi dx \int_\Omega v dx = (q, \nabla v)_{L_2(\Omega)} \;\forall\, v \in H^1(\Omega). \tag{2.1}$$

Show that the BVP (2.1) has a unique solution $\varphi \in H^1(\Omega)$ satisfying the orthogonality condition $(\varphi, 1)_{L_2(\Omega)} = 0$, i.e. $\varphi \perp \mathbf{R}$ in $L_2(\Omega)$!

17 Let us again consider the BVP described in 16. Show that $q - \nabla\varphi \in H(div, \Omega)$, $\mathrm{div}(q - \nabla\varphi) = 0$ in $[L_2(\Omega)]^3$ and $\mathrm{tr}_n(q - \nabla\varphi) = 0$ in $H^{-1/2}(\Gamma)$!

18* Show that the bilinear form $b(\cdot,\cdot) : H_0(curl, \Omega) \times H_0^1(\Omega) \to \mathbf{R}$ defined by the identity

$$b(v, \varphi) = \int_\Omega v \cdot \nabla\varphi \, dx \;\; \forall\, v \in H_0(curl, \Omega),\; \varphi \in H_0^1(\Omega) \tag{2.2}$$

fulfils the so-called LBB-condition, i.e. there is a positive constant β_1 such that

$$\sup_{v \in H_0(curl,\Omega)} \frac{b(v, \varphi)}{\|v\|_{H(curl)}} \geq \beta_1 \|\varphi\|_{H^1(\Omega)} \;\forall\, \varphi \in H_0^1(\Omega) \;! \tag{2.3}$$

19 Let $\kappa(\cdot)$ be a real function from $L_\infty(\Omega)$ such that $0 < \kappa_1 \leq \kappa(x) \leq \kappa_2$ for all almost $x \in \Omega$ with some positive constants κ_1 and κ_2. Derive the Variational Formulation (VP) of the mixed BVP

$$\begin{aligned} \mathrm{curl}(\nu\, \mathrm{curl}(u)) + \kappa u &= J_i - \mathrm{curl}(M) \;\text{ in } \Omega, \\ u \times n &= g_D \;\text{ on } \Gamma_D, \\ \nu\, \mathrm{curl}(u) \times n &= g_N \;\text{ on } \Gamma_N, \end{aligned}$$

and formulate practically relevant conditions for the data ν, κ, J_i, M, g_D and g_N such that the VP has a unique solution, i.e. verify the conditions of the Lax-Milgram-Theorem !