T U T O R I A L

“Computational Electromagnetics”

to the lecture

“Numerical Methods in Electrical Engineering”

Tutorial 04 Thursday, May 06, 2010 (Time: 15:30 – 16:15; Room: T 212)

13 Let $\Omega_1, \dots, \Omega_m$ be a non-overlapping domain decomposition of Ω, i.e. $\overline{\Omega} = \cup \overline{\Omega}_i$, $\Omega_i \cap \Omega_j = \emptyset$, $i \neq j$. Let $u_i \in H(curl, \Omega_i)$, $i = 1, 2, \dots, m$, such that

$$\mathrm{tr}_{t_i,\Gamma_{ij}} u_i = \mathrm{tr}_{t_i,\Gamma_{ij}} u_j \;\; \forall\, \Gamma_{ij} = \partial\Omega_i \cap \partial\Omega_j \;:\; \mathrm{meas}_{d-1}\Gamma_{ij} > 0.$$

Then the piecewise defined function

$$u := \{u|_{\Omega_i} = u_i,\, i = 1, 2, \dots, m\} \in H(curl, \Omega) \text{ and } (\mathrm{curl} u)|_{\Omega_i} = \mathrm{curl} u_i,$$

for all $i = 1, 2, \dots, m$.

14 Show that the inf-norm

$$|||q_n|||_{H^{-1/2}(\Gamma)} = \inf_{q \in H(div,\Omega):\, tr_n q = q_n} ||q||_{H(div)} \quad \forall\, q_n \in H^{-1/2}(\Gamma) \tag{2.1}$$

is equivalent to the usual $H^{-1/2}$-norm defined by the relation

$$||q_n||_{H^{-1/2}(\Gamma)} = \sup_{w \in H^{1/2}(\Gamma)} \frac{\langle q_n, w\rangle_{H^{-1/2}(\Gamma)\times H^{1/2}(\Gamma)}}{||w||_{H^{1/2}(\Gamma)}} \quad \forall\, q_n \in H^{-1/2}(\Gamma). \tag{2.2}$$

15 Show the second part of Theorem 2.5, i.e. show that, for a given $q_n \in H^{-1/2}(\Gamma)$ satisfying the relation $\langle q_n, 1\rangle_{H^{-1/2}(\Gamma)\times H^{1/2}(\Gamma)} = 0$, there exists an extension $q \in H(div)$ such that

$$tr_n q = q_n,\; \mathrm{div} q = 0 \text{ and } ||q||_{H(div)} \leq ||q_n||_{H^{-1/2}(\Gamma)}. \tag{2.3}$$

Hint: Consider the auxiliary Neumann problem $-\Delta u = 0$ in Ω and $\frac{\partial u}{\partial n} = q_n$ on $\Gamma = \partial\Omega$.