T U T O R I A L

"Computational Electromagnetics"

to the lecture

"Numerical Methods in Electrical Engineering"

Tutorial 03 Thursday, April 29, 2010 (Time: 15:30 – 16:15; Room: T 212)

2 The Variational Framework

2.1 Function Spaces and Properties.

09 Show that if there exists a week curl $c = \mathrm{curl}(u) \in [L_2(\Omega)]^3$ of a vector function $u \in [L_2(\Omega)]^3$ in the sense of Definition 2.1 then c is uniquely defined !

10 Show that if $g = \nabla w \in [L_2(\Omega)]^3$ is the weak gradient of a scalar function $w \in L_2(\Omega)$ in the sense of Definition 2.1 then $g \in H(\mathrm{curl})$ and the weak curl of g is 0, i.e. $\mathrm{curl}(g) = \mathrm{curl}\nabla w = 0$ in $[L_2(\Omega)]^3$!

11 Show that, for sufficiently smooth functions, e.g. for $u, v \in H(curl) \cap [C^1(\overline{\Omega})]^3$, the curl-IbyP-formula

$$\int_\Omega \mathrm{curl}(u) \cdot v \, dx = \int_\Omega u \cdot \mathrm{curl}(v) \, dx - \int_\Gamma (u \times n) \cdot v \, ds \qquad (2.1)$$

is valid. **Hint:** Use the classical IbyP-formula for the proof of (2.1) !

12 Let $\Omega_1, \dots, \Omega_m$ be a non-overlapping domain decomposition of Ω, i.e. $\overline{\Omega} = \cup \overline{\Omega}_i$, $\Omega_i \cap \Omega_j = \emptyset,\ i \neq j$. Let $q_i \in H(div, \Omega_i)$, $i = 1, 2, \dots, m$, such that

$$\mathrm{tr}_{n_i,\Gamma_{ij}} q_i = \mathrm{tr}_{n_i,\Gamma_{ij}} q_j \ \ \forall\, \Gamma_{ij} = \partial\Omega_i \cap \partial\Omega_j \ :\ \mathrm{meas}_{d-1}\Gamma_{ij} > 0.$$

Then the piecewise defined function

$$q := \{q|_{\Omega_i} = q_i,\ i = 1, 2, \dots, m\} \in H(div, \Omega) \text{ and } (\mathrm{div} q)|_{\Omega_i} = \mathrm{div} q_i,$$

for all $i = 1, 2, \dots, m$.

13 Let $\Omega_1, \dots, \Omega_m$ be a non-overlapping domain decomposition of Ω, i.e. $\overline{\Omega} = \cup \overline{\Omega}_i$, $\Omega_i \cap \Omega_j = \emptyset,\ i \neq j$. Let $u_i \in H(curl, \Omega_i)$, $i = 1, 2, \dots, m$, such that

$$\mathrm{tr}_{t_i,\Gamma_{ij}} u_i = \mathrm{tr}_{t_i,\Gamma_{ij}} u_j \ \ \forall\, \Gamma_{ij} = \partial\Omega_i \cap \partial\Omega_j \ :\ \mathrm{meas}_{d-1}\Gamma_{ij} > 0.$$

Then the piecewise defined function

$$u := \{u|_{\Omega_i} = u_i,\ i = 1, 2, \dots, m\} \in H(curl, \Omega) \text{ and } (\mathrm{curl} u)|_{\Omega_i} = \mathrm{curl} u_i,$$

for all $i = 1, 2, \dots, m$.