

Riesz-Schauder-Theory and Fredholm's Alternative

Theorem B5:

Let X be a normed space and $K: X \rightarrow X$.
The spectrum $\sigma(K)$ of a compact operator $K \in L_c(X) \subset L(X) := L(X, X)$ consists of at most countable many points $\{\lambda_1, \lambda_2, \dots\} \subset \mathbb{C}$ which can only accumulate at 0.

If X is infinite-dimensional then $0 \in \sigma(K)$.

$\lambda \in \sigma(K): \lambda \neq 0$ are eigenvalues.

The corresponding eigenspace $\text{Ker}(K - \lambda I)$ is finite-dimensional.

Theorem B6: Fredholm's Alternative

Ass: $K \in L(X)$ -compact, i.e. $K \in L_c(X)$,

$A = I - K \in L(X)$,

X -normed space, $\|\cdot\|_X$

St: $\text{im } A = (\text{Ker } A^*)^\perp$, i.e. the operator equation

Find $u \in X: (I - K)u = b$ in X

is solvable iff $\langle y^*, b \rangle = 0 \forall y^* \in \text{Ker } A^*$.

PROOF: see Numerik I