Theorem 3.21:

A: Let us assume that

- the standard assumptions (20) are fulfilled,
- and the problem is s-regular with some $s \in (0,1]$, cf. Section 2.2.5 (T 10 + 12), i.e. $u \in H^s(\mathrm{curl})$ and $\|u\|_{H^s(\mathrm{curl})} \le c\,\|f\|_{L_2}$.

S: Then the discretization error estimate

$$(21) \quad \|u - u_h\|_{H(\mathrm{curl})} \le c\,h^s\,\|f\|_{L_2}$$

is valid.

Proof: $V = H(\mathrm{curl})$, $u \in H^s(\mathrm{curl}) \cap C^0$

$$\|u-u_h\|_V \le \|u - I_h^E u\|_V + \|I_h^E u - u_h\|_V \le$$

$$\le \|u - I_h^E u\|_V + \tilde{\mu}_1^{-1} \sup_{v_h \in \tilde{V}_h} \frac{a(I_h^E u - u_h, v_h)}{\|v_h\|_V}$$

$$= \|u - I_h^E u\|_V + \tilde{\mu}_1^{-1} \sup_{v_h \in \tilde{V}_h} \frac{a(I_h^E u - u, v_h)}{\|v_h\|_V}$$

$a(u_h, v_h) = a(u, v_h) \;\forall v_h \in \tilde{V}_h \subset \tilde{V}$

$$\le \|u - I_h^E u\|_V + \frac{\mu_2}{\tilde{\mu}_1} \|I_h^E u - u\|_V =$$

$$= \left(1 + \frac{\mu_2}{\tilde{\mu}_1}\right) \|u - I_h^E u\|_V$$

$$\le \left(1 + \frac{\mu_2}{\tilde{\mu}_1}\right) c\,h^s\,\|u\|_{H^s(\mathrm{curl})}$$

$$\le \left(1 + \frac{\mu_2}{\tilde{\mu}_1}\right) c\,h^s\,\|f\|_{L_2(\Omega)} . \quad \text{q.e.d.}$$