3.4. Discretization Error Estimate

- Let us consider an $H(\mathrm{curl})$-VP of the form

(19) Find $u \in \tilde{V}$: $a(u,v) = \langle F, v \rangle \quad \forall v \in \tilde{V}$

and its FE-Galerkin Approximation

$(19)_h$ Find $u_h \in \tilde{V}_h$: $a(u_h, v_h) = \langle F, v_h \rangle \quad \forall v_h \in \tilde{V}_h$,

for instance:

(5) = (2.15) Find $u \in V = \tilde{V}_0 = H_0(\mathrm{curl})$:

$$\underbrace{\int_\Omega (\nu \,\mathrm{curl}\, u \cdot \mathrm{curl}\, v + \varkappa u \cdot v)\, dx}_{= a(u,v)} = \underbrace{\int_\Omega f \cdot v\, dx}_{= \langle F, v \rangle} \quad \forall v \in \tilde{V}$$

with $\nu, \varkappa \in L_\infty(\Omega)$: $0 < \underline{\nu} \le \nu(x)$, $0 < \underline{\varkappa} \le \varkappa(x)$ $\forall x \in \Omega$, ...

OR

(2.22) = eddy current problem in the frequency domain

Find $u = (u_r, u_i) \in V = \tilde{V}_0 = H_0(\mathrm{curl}) \times H_0(\mathrm{curl})$:

$$a(u,v) = \langle F, v \rangle \quad \forall v \in \tilde{V}$$

with

$$a(u,v) = \int_\Omega [\nu(\mathrm{curl}\, u_r \cdot \mathrm{curl}\, v_r + \mathrm{curl}\, u_i \cdot \mathrm{curl}\, v_i) + \varkappa_r (u_r v_r + u_i v_i) + \varkappa_i (u_r v_i - u_i v_r)]\, dx,$$

$$\langle F, v \rangle = \int_\Omega (f_r v_r + f_i v_i)\, dx, \quad f_r, f_i \in L_2(\Omega)^3.$$

under the assumptions

(20)
1) $F \in V^*$

2a) inf-sup
$$\inf_{u \in V} \sup_{v \in \tilde{V}} \frac{a(u,v)}{\|u\|_V \|v\|_V} \ge \mu_1 > 0$$

$2a)_h$ discrete inf-sup
$$\inf_{u_h \in V_h} \sup_{v_h \in \tilde{V}_h} \frac{a(u_h, v_h)}{\|u_h\|_V \|v_h\|_V} \ge \tilde{\mu}_1 > 0$$

2b) $|a(u,v)| \le \mu_2 \|u\|_V \|v\|_V \quad \forall u, v \in \tilde{V}$