

■ Def. 3.19: Raviart-Thomas FE

The lowest order tetrahedral Raviart-Thomas FE is given by

- a tetrahedron T ,
- the local FE-space $\mathcal{RT}_0 := \{v = a + bx : a \in \mathbb{R}^3, b \in \mathbb{R}\}$,
- the functionals

$$\Psi_{F_{\alpha\beta\gamma}} : v \longrightarrow \int_{F_{\alpha\beta\gamma}} v \cdot n \, ds$$

associated with the 4 faces $F_{\alpha\beta\gamma}$ of the tetrahedron T .

■ Properties 3.20: of the Raviart-Thomas FE-space

1. The nodal basis function $\Psi_{\alpha\beta\gamma}$ associated with the face $F_{\alpha\beta\gamma}$ of the tetrahedron T is given by

$$(18) \quad \Psi_{\alpha\beta\gamma} = \lambda_\alpha \nabla \lambda_\beta \times \nabla \lambda_\gamma + \lambda_\beta \nabla \lambda_\gamma \times \nabla \lambda_\alpha + \lambda_\gamma \nabla \lambda_\alpha \times \nabla \lambda_\beta.$$

$$2. \quad [P_0]^3 \subset \mathcal{RT}_0 \subset [P_1]^3$$

$$3. \quad \operatorname{div} \mathcal{RT}_0 = P_0$$

4. The normal components on the faces are constant.

$$5. \quad \operatorname{curl} \mathcal{N}_0 = \{v \in \mathcal{RT}_0 : \operatorname{div} v = 0\}$$

$$6. \quad \mathcal{R}_h := \{v \in L_2(\Omega)^3 : v|_T \in \mathcal{RT}_0 \wedge (\Psi_{F_{\alpha\beta\gamma}} \equiv \Psi_{\hat{F}_{\alpha\beta\gamma}})$$

$$\Rightarrow \Psi_{F_{\alpha\beta\gamma}}(v|_T) = \Psi_{\hat{F}_{\alpha\beta\gamma}}(v|_{\hat{T}})\} \subset H(\operatorname{div}).$$