

■ Def. 3.17: Edge FE = Nédélec FE

The lowest order tetrahedral edge (Nédélec) FE is given by

- a tetrahedron T
- the local FE space $\mathcal{N}_0 := \{v = a + b \cdot x : a, b \in \mathbb{R}^3\}$
- the functionals

$$\psi_{E_{\alpha\beta}} : v \mapsto \int_{E_{\alpha\beta}} v \cdot t \, ds$$

associated with the 6 edges $E_{\alpha\beta}$ of the tetrahedron T .

■ Properties 3.18: of the edge FE-space

1. As in 2D, the nodal basis function $\varphi_{\alpha\beta}$ associated with the edge $E_{\alpha\beta}$ is given by

$$(17) \quad \varphi_{\alpha\beta} = \lambda_\alpha \nabla \lambda_\beta - \lambda_\beta \nabla \lambda_\alpha.$$

2. The tangential trace of all $v \in \mathcal{N}_0$ onto a face F of ∂T exactly gives the 2D Nédélec triangle.

$$3. \quad \tilde{V}_h := \left\{ v \in L_2(\Omega)^3 : v|_T \in \mathcal{N}_0 \wedge (\psi_{E_{\alpha\beta}} \equiv \psi_{\tilde{E}_{\alpha\beta}} \Rightarrow \psi_{E_{\alpha\beta}}(v|_T) = \psi_{\tilde{E}_{\alpha\beta}}(v|_F)) \right\} \subset H(\text{curl})$$

4. $\forall v_h \in \tilde{V}_h$: $\text{curl } v_h$ is piecewise constant!

5. $\text{curl } \tilde{V}_h \subset H(\text{div})$, i.e.

$\forall v_h \in \tilde{V}_h$; the normal component of $\text{curl } v_h$ is continuous across the faces!

6. Interpolation error estimates:

$$(18) \quad \|u - I_h^E u\|_{H(\text{curl})} \leq ch \left(\|u\|_{H^1}^2 + \|\text{curl } u\|_{H^1}^2 \right)^{1/2}$$

$$\leq ch \|u\|_{H^1(\text{curl})}$$

$$\|u - I_h^E u\|_{H(\text{curl})} \leq ch^S \|u\|_{H^S(\text{curl})}, \quad S \in [0, 1].$$