- <u>Theorem 3.15:</u> The commuting identity

$$(15) \quad \operatorname{curl} I_h^E u = I_h^T \operatorname{curl} u$$

holds for all $u \in H(\operatorname{curl}) \cap C^0$.

<u>Proof:</u> mms

- <u>Theorem 3.16:</u>
The interpolation error estimate

$$(16) \quad \| \operatorname{curl} (u - I_h^E u) \|_{L_2(T)} \le ch \, |\operatorname{curl} u|_{H^1(T)}$$

is valid for all $u \in H(\operatorname{curl}) \cap C^0$ with $\operatorname{curl} u \in H^1$.

<u>Proof:</u>

1) By scaling and a Bramble-Hilbert argument, we can easily prove (mms)

$$\| s - I_h^T s \|_{L_2(T)} \le ch \, |s|_{H^1(T)} \quad \forall s \in H^1(T).$$

2) Now, using identity (15), we obtain

$$\| \operatorname{curl} (u - I_h^E u) \|_{L_2(T)} \overset{(15)}{=} \| (I - I_h^T) \underbrace{\operatorname{curl} u}_{\in H^1} \|_{L_2(T)}$$

$$\le ch \, |\operatorname{curl} u|_{H^1(T)}.$$

q.e.d.

- Th. 3.13 + Th. 3.16 yields

$$(17) \quad \| u - I_h^E u \|_{H(\operatorname{curl})} \le ch \left(|u|_{H^1}^2 + |\operatorname{curl} u|_{H^1}^2 \right)^{1/2}.$$