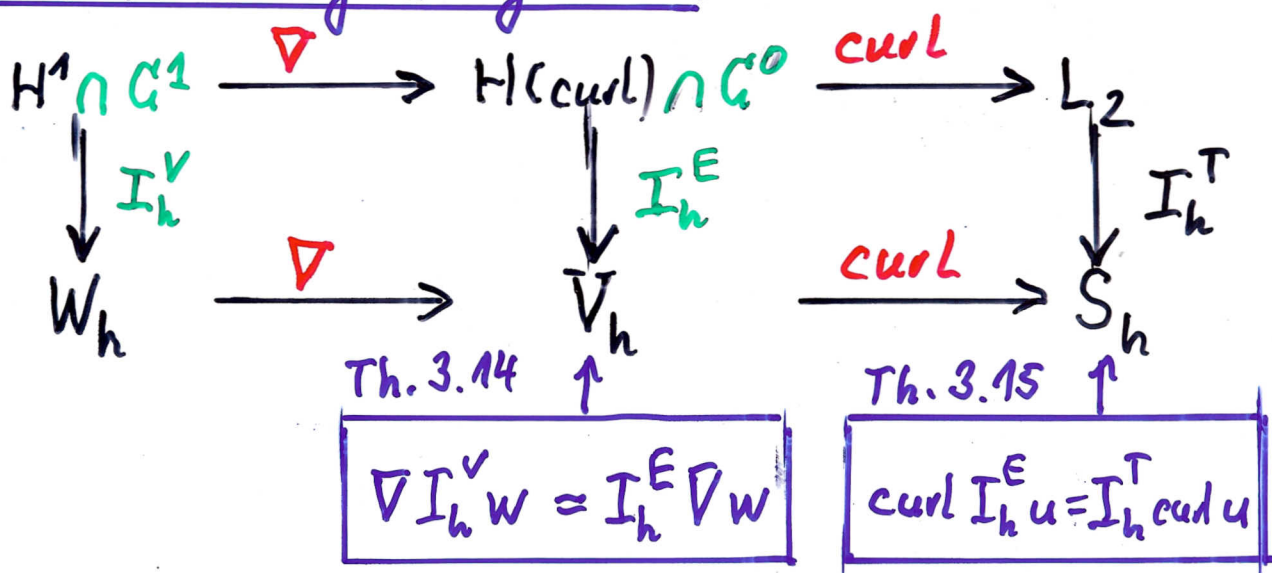


■ Commuting diagram:



where I_h^V - vertex interpolation operator (\uparrow),
 I_h^E - edge interpolation operator (\uparrow),
 I_h^T - element interpolation operator onto piecewise constant elements with the functional (dof) $\psi_T(s) = \int_T s \, dx$

■ Theorem 3.14: The commuting identity
 (14) $\nabla I_h^V w = I_h^E \nabla w$
 holds for all $w \in H^1 \cap C^1$.

Proof: 1) $\nabla I_h^V w \in \tilde{V}_h$ due to Theorem 3.9.

2) $I_h^E \nabla w \in \tilde{V}_h$ (trivial!)

3) It remains to show that $\psi_{E_{ij}}(\nabla I_h^V w) \stackrel{!}{=} \psi_{E_{ij}}(I_h^E \nabla w)$!

$$3a) \psi_{E_{ij}}(\nabla I_h^V w) = \int_{E_{ij}} \nabla I_h^V w \cdot t \, ds = (I_h^V w)(V_j) - (I_h^V w)(V_i) = w(V_j) - w(V_i)$$

$$3b) \psi_{E_{ij}}(I_h^E \nabla w) = \psi_{E_{ij}}\left(\sum_{E_{k\ell}} \psi_{E_{k\ell}}(\nabla w) \psi_{E_{k\ell}}\right)$$

$$\psi_{E_{ij}}(\psi_{E_{k\ell}}) = \delta_{ij, k\ell} \Rightarrow \psi_{E_{ij}}(\nabla w) = \int_{E_{ij}} \nabla w \cdot t \, ds = w(V_j) - w(V_i)$$

q.e.d.