Theorem 3.13:

The Nédélec (edge) interpolation operator $I_h := I_h^E$ satisfies the following L_2-estimate:

$$(13)\quad \|u - I_h u\|_{L_2(T)} \leq c\, h\, |u|_{H^1(T)} \quad \forall u \in H(\mathrm{curl}) \cap (H^1)^2.$$

Proof: We transform T to the reference element T^R, and define

$$u^R(\xi) := J_T^T\, u(\Phi_T(\xi)).$$

Then we have $T \to T^R$, (12)

$$\|u - I_h u\|^2_{L_2(T)} =$$

$$= \int_{T^R} |u(\Phi_T(\xi)) - J^{-T} I_h^R (J^T u(\Phi_T(\xi)))|^2 \, |\det J| \, d\xi$$

$$= \int_{T^R} | J^{-T} (u^R(\xi) - I_h^R(u^R(\xi)))|^2 \, |\det J| \, d\xi$$

$\|J^{-T}\| \lesssim h^{-1}$, $|\det J| \leq h^2$ see NuEPDE

$$\lesssim \|u^R - I_h^R u^R\|^2_{L_2(T^R)}$$

$$\lesssim |u^R|^2_{H^1(T^R)} \overset{(mms)}{\lesssim} h^2 |u|^2_{H^1(T)}$$

Bramble-Hilbert-Lemma (mms) see also NuEPDE

1) $(I - I_h^R)c = 0 \quad \forall c = \binom{a_1}{a_2}$ with $a_1, a_2 \in \mathbb{R}$

   Remark: $\binom{a_1}{a_2} \in \mathcal{N}_0$; $c = \binom{1}{0}$ and $c = \binom{0}{1}$ is enough!

2) $I - I_h^R \in L(H^1, L_2)$, i.e. show that $I_h^R \in L(H^1, L_2)$,

   i.e. $\left|\int_{E^R_{\alpha\beta}} u^R \cdot t \, ds\right| \leq c \|u^R\|_{H^1(T^R)}$

   H¹-trace theorem

q.e.d.