Lemma 3.12:

If we use the covariant transformation, then the interpolation on an arbitrary Nédélec element T is equivalent to the corresponding interpolation on the reference element T^R:

$$(12) \quad I_h^R[\mathcal{J}^T u \circ \Phi] = \mathcal{J}^T (I_h u) \circ \Phi$$

Proof:

$$\sum_{E_{\alpha\beta} \subset \partial T^R} \underbrace{\psi_{\alpha\beta}^R (\mathcal{J}^T u \circ \Phi)} \varphi_{\alpha\beta}^R \overset{!}{=} \mathcal{J}^T \sum_{E_{\alpha\beta} \subset \partial T} \underbrace{\psi_{\alpha\beta}(u)} \underbrace{\varphi_{\alpha\beta} \circ \Phi}_{= \varphi_{\alpha\beta}^R}$$

$$\psi_{\alpha\beta}^R (\mathcal{J}^T u \circ \phi) \overset{!}{=} \psi_\alpha (u)$$

$$\int_{E^R} \mathcal{J}^T u(\phi(\xi)) \cdot t^R \, ds_\xi = \int_{\Phi(E^R)} u \cdot t \, ds_x$$

This relation is also true for general curves. Indeed, let E^R be parametrized with

$$\gamma : [0,1] \longrightarrow \mathbb{R}^2$$

Then we have

$$\int_0^1 [\mathcal{J}^T u(\phi(\gamma(s)))] \cdot \gamma'(s) \, ds \overset{!}{=} \int_0^1 u(\phi(\gamma(s))) \cdot [\phi(\gamma(s))]' \, ds$$

$$= \int_0^1 u(\phi(\gamma(s))) \cdot \mathcal{J} \gamma'(s) \, ds$$

q.e.d.