3.2.4. Interpolation Operators and Interpolation Error Estimates

- "Nodal" Interpolation Operator: $\psi_i(\varphi_j) = \delta_{ij}$

(10) $$I_h u(x) := \sum_{i=1}^{N_h} \psi_i(u)\,\varphi_i(x)$$

- Lemma 3.11: $I_h^2 = I_h$

  i.e. I_h is a projector.

  Proof:

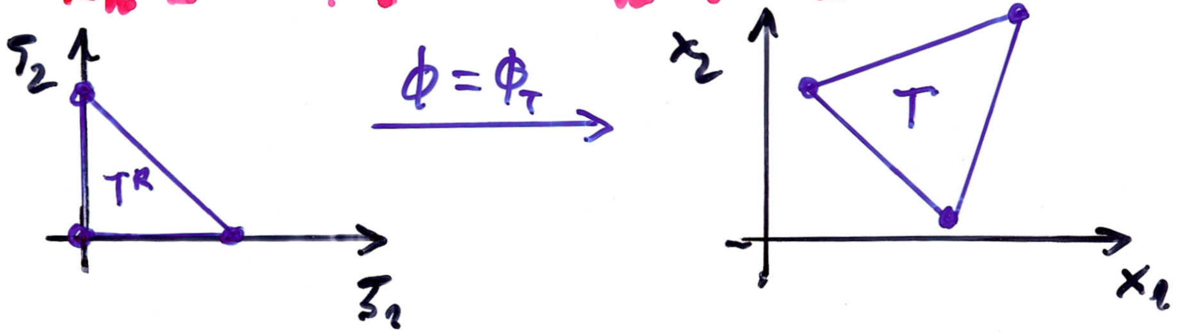
  $$I_h I_h u = \sum_{j=1}^{N} \psi_j\Big(\sum_{i=1}^{N} \psi_i(u)\,\varphi_i\Big)\varphi_j$$

  $$= \sum_{j=1}^{N}\sum_{i=1}^{N} \psi_i(u)\,\underbrace{\psi_j(\varphi_i)}_{=\delta_{ij}}\,\varphi_j = \sum_{i=1}^{N}\psi_i(u)\varphi_i = I_h u$$

  q.e.d.

- Interpolating a function on an element T should result in the same function as the interpolation on the reference element T^R: This is trivial for nodal elements:

(11) $$I_h^R[u\circ\Phi] = (I_h u)\circ\Phi$$



! This is not true for curved element, which needs a more technical analysis!