3.2.3. Implementation Aspects

- NuPDE + NuEPDE: Assembling process

The stiffness matrix

$$K_h = A + M = [A_{ij}] + [M_{ij}], \quad A_{ij} = \int_\Omega \nu \operatorname{curl} \varphi_j \cdot \operatorname{curl} \varphi_i \, dx,$$

i and j are associated with the edges of the mesh

$$M_{ij} = \int_\Omega \varkappa \, \varphi_j \cdot \varphi_i \, dx,$$

and the rhs

$$\underline{f}_h = [f_i] \text{ with } f_i = \int_\Omega f \cdot \varphi_i \, dx$$

of the system $(\underline{5})_h$ are assembled from the element contributions

$$A = \sum_T{}' C_T \underbrace{A_T}_{3\times 3} C_T^T, \quad M = \sum_T{}' C_T \underbrace{M_T}_{3\times 3} C_T^T \text{ and } \underline{f} = \sum_T{}' C_T \underline{f}_T,$$

$N_h [\,]^3$ $\mathbb{R}^3$

where C_T are the $N_h \times 3$ connectivity matrices connecting the local with the global numbering of the basis functions; $N_h = N_E$ = number of edges which do not belong to Γ_D.

We note that the orientation of the edges must be taken into account: If the local edge is opposite to the global one, then the entry in C is set to -1 !

- Computation of the element matrices A_T and M_T:

$$M_{T,k\ell} = \int_T \varkappa \, \varphi_k \cdot \varphi_\ell \, dx \overset{T\to T_R}{=} \int_{T_R} \varkappa \, (J_T^{-T} \varphi_k^R) \cdot (J_T^{-T} \varphi_\ell^R) \det J_T \, d\xi \;{*}\,\Sigma,$$

$$A_{T,k\ell} = \int_T \nu(x) \operatorname{curl}_x \varphi_k(x) \cdot \operatorname{curl}_x \varphi_\ell(x) \, dx \underset{x = \Phi_T(\xi)}{\overset{T\to T_R}{=}}$$

$$= \int_{T_R} \nu(\Phi_T(\xi)) (\det J_T)^{-1} \operatorname{curl}_\xi \varphi_k^R \cdot (\det J_T)^{-1} \operatorname{curl}_\xi \varphi_i^R \det J_T \, d\xi$$

$*$ Σ .... computed by a suitable integration rule!

Analogous: $f_{T,k} = \int_T f \cdot \varphi_k \, dx = \ldots$