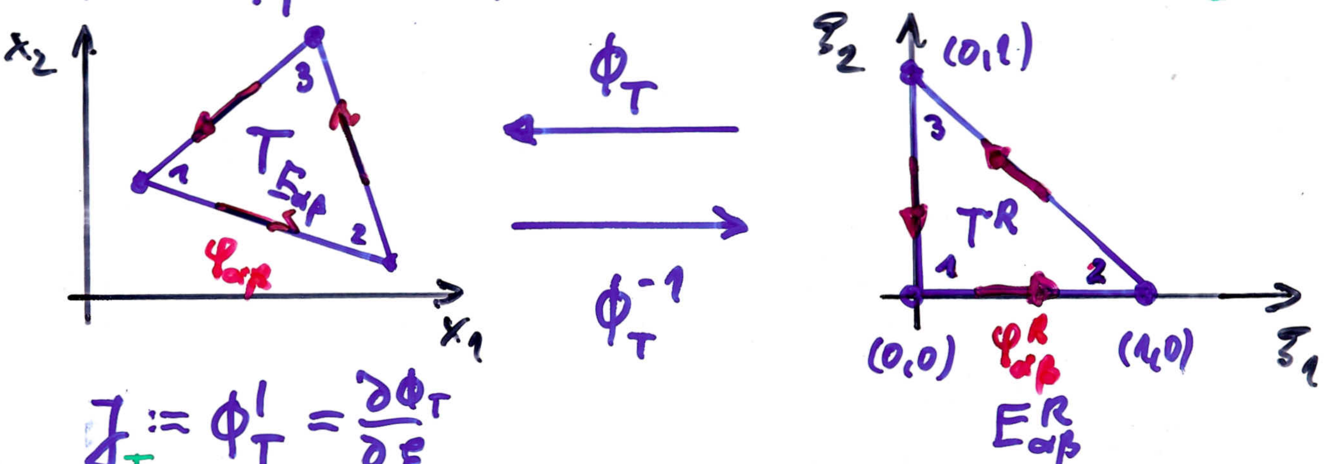
3.2.2. Reference Element and Mapping Principle

NuEPDE

- $T = \phi_T(T^R)$



$$J_T := \phi_T' = \frac{\partial \phi_T}{\partial \xi}$$

- <u>Lemma 3.10:</u>

Let $E^R_{\alpha\beta}$ be an edge of the reference element T^R, and $E_{\alpha\beta}$ the according edge of the general element T. Then the corresponding edge basis fct $\varphi^R_{\alpha\beta}$ and $\varphi_{\alpha\beta}$ satisfy

$$(9) \quad \varphi_{\alpha\beta} = J^{-T}\varphi^R_{\alpha\beta}, \quad \operatorname{curl}_x \varphi_{\alpha\beta} = \frac{1}{\det J}\operatorname{curl}_\xi \varphi^R_{\alpha\beta}.$$

This transformation is called covariant.

<u>Proof:</u> $\lambda_\alpha = \varphi_\alpha$ = vertex basis functions

∇
- $\lambda_\alpha(\phi_T(\xi)) = \lambda^R_\alpha(\xi) \quad \forall \xi \in T^R$

$\mapsto$
- $J^T(\nabla_x \lambda_\alpha)(\phi_T(\xi)) = \nabla_\xi \lambda^R_\alpha(\xi)$
- Now the edge-shape function in $x \in T$ is

$$\begin{aligned}\varphi_{\alpha\beta}(x) &= \lambda_\alpha(x)\nabla_x\lambda_\beta(x) - \lambda_\beta(x)\nabla_x\lambda_\alpha(x) \\ &= \lambda^R_\alpha(\xi)J^{-T}\nabla_\xi\lambda^R_\beta(\xi) - \lambda^R_\beta(\xi)J^{-T}\nabla_\xi\lambda^R_\alpha(\xi) \\ &= J^{-T}\varphi^R_{\alpha\beta}(\xi)\end{aligned}$$

- The proof of the transformation of the curls is based on

$$\operatorname{curl}[J^T u(\phi(x))] = (\det J)(\operatorname{curl} u)(\phi(x)) \qquad u = \varphi_{\alpha\beta} \ \blacksquare$$