2.2.5. Regularity Theory for Maxwell's Eqns

- **Def. 2.18:** (s-regularity for Poisson's Eqn.)
  The Poisson Eqn

  (29) $-\Delta\phi = f$ in Ω and $\phi = 0$ on $\Gamma = \partial\Omega$

  is called s-regular if $f \in L_2(\Omega)$ implies $\phi \in H^{1+s}(\Omega)$ with the regularity est.

  (30) $\|\phi\|_{H^{1+s}(\Omega)} \le c\, \|f\|_{L_2(\Omega)}$

- **Remark:**
  If Ω is either convex or smooth, then the Poisson Eqn is regular with $s = 1$.
  On Lip-domains, regularity holds with some $s \in (0,1)$.

- **Lemma 2.19:**
  Assume that the Poisson problem is s-regular.
  Let either $u \in H_0(curl) \cap H(div)$ or $u \in H(curl) \cap H_0(div)$.
  Then there holds

  (31) $\|u\|_{H^s(\Omega)} \le c\,(\|curl\, u\|_0 + \|div\, u\|_0$

  Remark: Note that one BC is really necessary.
  Indeed, take some non-const. harmonic function ϕ, i.e. $\Delta\phi = 0$, and set $u = \nabla\phi$.
  Then $div\, u = 0$ and $curl\, u = 0$, but $\|u\|_{H^1(\Omega)} > 0$.