Theorem 2.13: (Friedrichs-type inequalities)

1. Assume that $u \in H(\mathrm{curl})$ satisfies

$$(u, \nabla \psi)_{L_2} = 0 \quad \forall \psi \in H^1(\Omega).$$

Then the Friedrichs-type inequality

$$\| u \|_{L_2} \leq c \, \| \mathrm{curl}\, u \|_{L_2}$$

is valid.

2. Assume that $u \in H_0(\mathrm{curl})$ satisfies

$$(u, \nabla \psi)_{L_2} = 0 \quad \forall \psi \in H_0^1(\Omega).$$

Then the Friedrichs-type inequality

$$\| u \|_{L_2} \leq c \, \| \mathrm{curl}\, u \|_{L_2}$$

holds.

Proof:

1. Theorem 2.12 a): $u = \nabla \varphi + z$ with $\varphi \in H^1$, $z \in [H^1]^3$

Since $z = u - \nabla \varphi$ and $(u, \nabla \varphi) = 0$

$u \perp \nabla \varphi$ in L_2,

we have

$$\| z \|_{L_2}^2 = \| u \|_{L_2}^2 + \| \nabla \varphi \|_{L_2}^2 ,$$

i.e.

$$\| u \|_{L_2} \leq \| z \|_{L_2} \leq \| z \|_{H^1} \overset{\text{Th. 2.12}}{\leq} c \, \| \mathrm{curl}\, u \|_{L_2} .$$

2. Theorem 2.12 b): .... (mms)

q.e.d.