▪ Recall: $H^{1/2}(\Gamma) := tr_\Gamma H^1(\Omega)$

Ω is $* \cap$ Lip

- The trace space $H^1(\Omega) = \overline{C^\infty(\Omega)}^{\|\cdot\|_{H^1(\Omega)}}$ is naturally defined by

$$H^{1/2}(\Gamma) := \{ tr_\Gamma u \in L_2(\Gamma) : u \in H^1(\Omega) \} \subset L_2(\Gamma)$$

equipped with the norm $(d=3)$

$$(1) \quad \|w\|^2_{H^{1/2}(\Gamma)} := \|w\|^2_{L_2(\Gamma)} + \int_\Gamma \int_\Gamma \frac{|w(x)-w(y)|^2}{|x-y|^2}\,dx\,dy = (w,w)_{H^{1/2}(\Gamma)}$$

and the corresponding scalar product $(\cdot,\cdot)_{H^{1/2}(\Gamma)}$ ($\to$ H-space), where the trace operator

$$tr_\Gamma u = \gamma_0 u = \gamma_D u = u|_\Gamma : H^1(\Omega) \to H^{1/2}(\Gamma)$$

is defined as follows:

1) $(tr_\Gamma u)(x) := u(x) \;\; \forall x \in \Gamma \;\; \forall u \in H^1(\Omega) \cap C(\overline{\Omega})$.
2) Prove that $\exists\, c = c_T = const > 0$:

$$(2) \qquad \|tr_\Gamma u\|_{H^{1/2}(\Gamma)} \le c_T \|u\|_{H^1(\Omega)} \;\; \forall u \in H^1(\Omega) \cap C(\overline{\Omega}).$$

3) Closure principle: Since $C^\infty(\overline{\Omega})$ is dense in $H^1(\Omega)$, $tr_\Gamma u$ is well defined for all $u \in H^1(\Omega)$ and (2) is valid for all $u \in H^1(\Omega)$.

We call (2) also trace theorem.

- The inverse trace theorem (= extension theorem) is also valid: $\forall w \in H^{1/2}(\Gamma)\; \exists u \in H^1(\Omega)$:

$$(3) \quad tr_\Gamma u = w \;\text{ and }\; \|u\|_{H^1(\Omega)} \le c_E \|w\|_{H^{1/2}(\Gamma)},$$

with some universial, positive constant $c_E = const > 0$.

- From (2) and (3), we immediately obtain

$$(4) \quad \frac{1}{c_T}\|w\|_{H^{1/2}(\Gamma)} \overset{(2)}{\le} \underbrace{\inf_{\substack{u \in H^1(\Omega) \\ tr_\Gamma u = w}} \|u\|_{H^1(\Omega)}}_{=: |||\cdot|||_{H^{1/2}} \simeq \|\cdot\|_{H^{1/2}}} \overset{(3)}{\le} c_E \|w\|_{H^{1/2}(\Gamma)} \quad \forall w \in H^{1/2}(\Gamma)$$