1.3. The General curl-curl - Equation as a Central Model Problem in EM

■ Let $\Omega \subset \mathbb{R}^{d=3}$ be a bounded Lip-domain with the boundary $\Gamma = \partial\Omega = \Gamma_D \cup \Gamma_N$, $\Gamma_D \cap \Gamma_N = \emptyset$.
Find $u = (u_1, u_2, u_3)^T : \overline{\Omega} \longrightarrow \mathbb{C}^3$ or $\mathbb{R}^3$:

(18)
$$\begin{aligned} &\operatorname{curl}(\nu \operatorname{curl}(u)) + \varkappa u = f \quad \text{in } \Omega \\ &+ \text{BC: } u \times n = g_D \text{ on } \Gamma_D = \Gamma_B, \\ &\qquad \nu \operatorname{curl} u \times n = g_N \text{ on } \Gamma_N = \Gamma_H \end{aligned}$$

where $u = A$ or E and

$\varkappa = 0$ – for the magnetostatic problem (15), Coulomb gauge (14) $\operatorname{div} u = 0$;

$\mathbb{R}^+ \ni \varkappa = \delta$ (small) – for the regularization;

$\mathrm{Im} \ni \varkappa = i\omega\delta$ – for the conductivity regularization;

$\mathbb{R} \ni \varkappa \approx \frac{\varepsilon}{\tau^2} + \frac{\sigma}{\tau}$ – for the implicit time discretization of $(9)^*_A$ or $(10)_E$, τ - time step;

$\mathbb{C}^- \ni \varkappa = -\varepsilon\omega^2 + i\omega\sigma$ – for the time-harmonic Maxwell eqns;

$\mathbb{R}^+_0 \ni \varkappa \approx \frac{\sigma}{\tau}$ – for an implicit time discretization of the eddy-current problem $(11)_{TD}$;

$\mathrm{Im} \ni \varkappa \approx i\omega\sigma$ – for the time-harmonic ECP $(11)_{FD}$;

$\mathbb{R}^+ \ni \varkappa \approx \varepsilon/\tau^2$ – for the impl. time discr. of the wave case;

$\mathbb{R}^- \ni \varkappa = -\varepsilon\omega^2$ – for the high-frequency time-harm. case.

↓

<u>In general</u>: $\varkappa = \varkappa(x)$, $x = (x_1, x_2, x_3) \in \Omega$

$\nu = \frac{1}{\mu} \approx \nu(x)$ OR $\nu = \nu(x, |\operatorname{curl}(\omega)|)$

↑ Linear case ↑ non-Linear