Vector, Differential and Integral Identities

1. Vector Identities:

$$a \times b = - b \times a$$
$$a \cdot (b \times c) = (a \times b) \cdot c = (c \times a) \cdot b$$

2. Differential Identities:

$$\nabla \times (\nabla p) = \text{curl}\, \nabla p = 0$$
$$\nabla \cdot (\nabla \times v) = \text{div curl}\, v = 0$$
$$\nabla \cdot (\phi v) = \nabla \Phi \cdot v + \phi \nabla \cdot v$$
$$\nabla \times (\phi v) = \Phi \nabla \times v + (\nabla \Phi) \times v$$
$$\nabla \times (u \times v) = u (\nabla \cdot v) - (u \cdot \nabla) v + (v \cdot \nabla) u - v (\nabla \cdot u)$$
$$\nabla \times (\nabla \times u) = \nabla (\nabla \cdot u) - \Delta u$$
$$\nabla \cdot (u \times v) = v \cdot \nabla \times u - u \cdot \nabla \times v$$

where $u = (u_1, u_2, u_3)^T$, $\Delta u = (\Delta u_1, \Delta u_2, \Delta u_3)^T$, $v = (v_1, v_2, v_3)^T$, $\phi = \phi(x)$ – scalar function.

3. Vector and Scalar Potentials:

$\text{curl}\, u = 0 \iff u = \nabla \varphi$, φ – scalar potential,
$\text{div}\, B = 0 \iff B = \text{curl}\, A$, A – vector potential,
in Ω provided that Ω is simply connected!

4. Integral Identities:

- GAUSS' theorem: $\displaystyle\int_V \text{div}\, u \, dx = \int_{\partial V} u \cdot n \, dS$
- STOKES' theorem: $\displaystyle\int_S \text{curl}\, u \cdot n \, dS = \oint_{\partial S} u \cdot \tau \, ds$