T U T O R I A L

“Numerical Methods for the Solution of Elliptic Partial Differential Equations”

to the lecture

“Numerical Methods for Elliptic Problems”

Tutorial 6 Tuesday, 5 May 2009 (Time : $10^{\underline{15}}$ – $11^{\underline{45}}$ Room : T 911)

2.4 Generation of the Finite Elements Equations

26 Show that the integration rule

$$\int_{\Delta} f(\xi,\eta)\, d\xi\, d\eta \approx \frac{1}{2}\{\alpha_1 f(\xi_1,\eta_1) + \alpha_2 f(\xi_2,\eta_2) + \alpha_3 f(\xi_3,\eta_3)\} \tag{2.22}$$

integrates quadratic polynomials exactly, if the the weights α_i and the integration points (ξ_i, η_i) are choosen as follows: $\alpha_1 = \alpha_2 = \alpha_3 = 1/3$ and $(\xi_1, \eta_1) = (1/2, 0)$, $(\xi_2, \eta_2) = (1/2, 1/2)$, $(\xi_3, \eta_3) = (0, 1/2)$.

27 Show that the integration rule

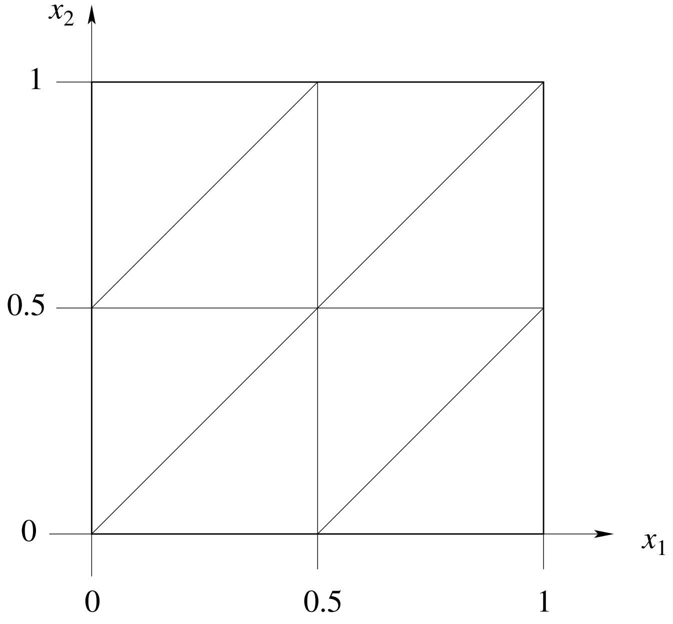
$$\int_{\Delta} f(\xi,\eta)\, d\xi\, d\eta \approx \frac{1}{2}\{\alpha_1 f(\xi_1,\eta_1) + \alpha_2 f(\xi_2,\eta_2) + \alpha_3 f(\xi_3,\eta_3)\} \tag{2.23}$$

integrates quadratic polynomials exactly, if the the weights α_i and the integration points (ξ_i, η_i) are choosen as follows: $\alpha_1 = \alpha_2 = \alpha_3 = 1/3$ and $(\xi_1, \eta_1) = (1/6, 1/6)$, $(\xi_2, \eta_2) = (4/6, 1/6)$, $(\xi_3, \eta_3) = (1/6, 4/6)$.

28 Generate the system of finite element equations for the mixed boundary value problem

$$-\Delta u(x_1,x_2) = 0 \quad \forall (x_1,x_2) \in \Omega := (0,1)\times(0,1), \tag{2.24}$$
$$u(x_1,0) = 0 \quad \forall x_1 \in [0,1], \tag{2.25}$$
$$u(0,x_2) = 0 \quad \forall x_2 \in [0,1], \tag{2.26}$$
$$\frac{\partial u}{\partial x_2}(x_1,1) = x_1 \quad \forall x_2 \in (0,1], \tag{2.27}$$
$$\frac{\partial u}{\partial x_1}(1,x_2) = x_2 \quad \forall x_2 \in (0,1], \tag{2.28}$$

using linear finite elements on the triangulation shown in the figure below. Solve the system of equations !



29* Extend the existing finite element software from the tutorials "Numerics of Partial Differential Equations" to the 2D, elliptic, second-order BVP (the heat conduction problem) given in Section 2.2.1 of our lecture ! Use linear triangular finite elements (Courant's elements) ! Choose a suitable solver for solving the linear FE-system $K_h \underline{u}_h = \underline{u}_h$ supporting the format, in which the sparse system matrix K_h is stored.

30* Solve the "Chip" problem with your finite element code developed in Exercise 29 !