T U T O R I A L

"Numerical Methods for the Solution of Elliptic Partial Differential Equations"

to the lecture

"Numerical Methods for Elliptic Problems"

Tutorial 5 Tuesday, 21 April 2009 (Time : $10^{\underline{15}}$ – $11^{\underline{45}}$ Room : T 911)

2 The Galerkin FEM

2.1 The Galerkin-Ritz method

22 Consider the following variational problem: Find $u \in V_g = V_0 = H_0^1(0,1)$ such that

$$\int_0^1 u'(x)\, v'(x)\, dx \;=\; \int_0^1 f(x)\, v(x)\, dx \qquad \forall v \in V_0. \tag{2.20}$$

Solve this variational problem by Galerkin's method using the basis

$$V_{0h} = V_{0n} = \operatorname{span}\big\{x(1-x),\, x^2(1-x),\, \dots,\, x^{n-1}(1-x)\big\},$$

where the right-hand side is given by $f(x) = \cos(k\pi x)$, $k = l+1$ and l is the last digit from your student code. Compute the stifness matrix K_h analytically and solve the linear system $K_h \underline{u}_h = \underline{f}_h$ numerically using Gaussian elimination (using `matlab`, `C`, or anything else). Consider n to be $2, 4, 8, 10, 50, 100$.

2.2 Mesh generation and refinement

23⋆ Consider a plane domain (2D) meshed into triangles. Such a mesh can be stored in the `*.net` format (see Silde 10). Implement an algorithm which inputs a given mesh file `coarse.net` and outputs the file `fine.net` containing the refinement of the coarse triangulation (each coarse triangle is subdivided into four fine triangles).

24⋆⋆ Modify the above algorithm such that one can select individual elements for refinement. The output mesh should meet the following requirements:

– selected elements are divided into four elements,

– neighboring elements that are not selected are divided into two or more elements (in order to guarantee the conformity),

– the remaining elements are not refined at all.

Later on, this algorithm can be used for an adaptive (a posteriori) error estimator as a part of a finite element program.

2.3 Mapping reference to physical element

24 Let δ_r denote a triangle, h_r its longest side and θ_r its smallest angle. Furthermore, let $|J_{\delta_r}|$ denote the Jacobi determinant of the affine linear mapping from the reference element to δ_r (see also your lecture notes). Show the inequality

$$\frac{\alpha_0^2}{2} \sin\theta_r \, h_r^2 \;\leq\; |J_{\delta_r}| \;\leq\; \frac{\sqrt{3}}{2} \, h_r^2 \,, \tag{2.21}$$

where the shortest side is of length $\alpha_0 \, h_r$.