T U T O R I A L

"Numerical Methods for the Solution of Elliptic Partial Differential Equations"

to the lecture

"Numerics of Elliptic Problems"

Tutorial 3 Tuesday, 24 March 2009 (Time : 10:15 – 11:45, Room : T 911)

1.3 Scalar Elliptic Problems of the Fourth Order

13 Show that the first biharmonic BVP

$$u \in V_0 := H_0^2(\Omega) : \int_\Omega \Delta u(x) \Delta v(x) dx = \int_\Omega f(x) v(x) dx \; \forall v \in V_0 \tag{1.12}$$

has a unique solution (Lax-Milgram-Theorem) ! Then, formulate a minimization problem that is equivalent to the variational formulation (1.12) !

14 Give the variational formulation for the second biharmonic BVP mentioned in Remark 1.6.2, and discuss the existence and uniqueness of generalized solutions ! Without loss of generality, consider homogenized essential boundary conditions only.

15* Give the variational formulations for the third and the fourth biharmonic BVPs mentioned in Remark 1.6.2, and discuss the existence and uniqueness of generalized solutions ! Without loss of generality, consider homogenized essential boundary conditions only.

16* For the Kirchhoff plate, the plate bilinear form

$$a(u,v) := \int_\Omega \left\{ \Delta u(x) \Delta v(x) + (1-\sigma) \left[2 \frac{\partial^2 u}{\partial x_1 \partial x_2} \frac{\partial^2 v}{\partial x_1 \partial x_2} - \frac{\partial^2 u}{\partial x_1^2} \frac{\partial^2 v}{\partial x_2^2} - \frac{\partial^2 u}{\partial x_2^2} \frac{\partial^2 v}{\partial x_1^2} \right] \right\} dx \tag{1.13}$$

is identical to the biharmonic bilinear form given in (1.12) for the first biharmonic BVP (i.e., on $H_0^2(\Omega)$) only. Prove this statement provided that $\sigma \in (0,1)$ is a given material parameter (Poisson coefficient). Which natural boundary can be impossed for the plate bilinear form (1.13) ?