## PA03 "PISTON": Head conduction

Compute the temperature field in the following body that is rotationally symmetric (the rotated area $\Omega$ is sketched below). On part $\Gamma_{1}$ of the boundary, the temperature is prescribed by $280^{\circ}$ C. On $\Gamma_{2}$ there holds $\frac{\partial u}{\partial n}=0$ due to the symmetry. On the rest, $\Gamma_{3}$, there occurs heat flux with the surrounding, the temperature outside is $20^{\circ} \mathrm{C}$.


The material parameters are given by $\lambda=0.159 \frac{\mathrm{~W}}{\mathrm{~mm} \cdot \mathrm{~K}}$ and $\alpha=4.06 \cdot 10^{-5} \frac{\mathrm{~W}}{\mathrm{~mm}^{2} \cdot \mathrm{~K}}$.

## Tasks:

- Derive the mathematical model. If possible use symmetries or reduction in the dimension
- Give the variational formulation
- Analysis: discuss existence and uniqueness of solutions
- Discretize the domain $\Omega$
- Numerical analysis: provide an error estimate
- Choose a solver for the system of equations you obtain
- Implementation
- Visualize the results and (if possible) compare them to analytical solutions


## 1 Mathematical model

2 Variational formulation
3 Analysis
4 Discretization
5 Solver
6 Numerical analysis
7 Implementation
8 Numerical results

