Questions for the Oral Examination

1. Derive the variational formulation of the second-order boundary value problem (5) given in Subsection 1.2.1, and show existence and uniqueness under some assumptions specified by the examiner !

2. Derive the variational formulation of the linear elasticity problem (8) given in Subsection 1.2.2, and show existence and uniqueness under some assumptions specified by the examiner !

3. Derive the variational formulation of the first biharmonic BVP (12) given in Subsection 1.2.3, and show existence and uniqueness of a weak solution in $H_0^2(\Omega)$! Which combinations of boundary conditions are possible ?

4. Electromagnetic fields are described by Maxwell's equations

$$\begin{aligned} \mathrm{curl} H &= J + \frac{\partial D}{\partial t}, \\ \mathrm{div} B &= 0, \\ \mathrm{curl} E &= -\frac{\partial B}{\partial t}, \\ \mathrm{div} D &= \varrho, \end{aligned}$$

with the constitutive relations

$$\begin{aligned} B &= \mu H + \mu_0 M \\ D &= \varepsilon E + P \\ J &= \sigma E + J_i \end{aligned}$$

and suitable interface, boundary and initial conditions. Derive the so-called eddy current equations

$$\sigma \frac{\partial A}{\partial t} + \mathrm{curl}(\frac{1}{\mu}\mathrm{curl}(A)) = J \ \text{ in } Q_T = \Omega \times (0, T)$$

in the vector potential ($B = \mathrm{curl}(A)$) formulation ! An implicit time integration, e.g. by the implicit Euler method, results in a boundary value problem (BVP) of the form

$$\begin{aligned} \mathrm{curl}(\frac{1}{\mu}\mathrm{curl}(u)) + \kappa u &= f \ \text{ in } \Omega, \\ u \times n &= 0 \ \text{ in } \partial\Omega \end{aligned}$$

that one has to solve in every time integration step with $\kappa = \sigma/\tau$ and an appropriate right-hand side f. Derive the right variational formulation,

$$\text{Find } u \in V_g : a(u, v) = \langle F, v \rangle, \ \forall v \in V_0,$$

of this boundary value problem, and show existence and uniqueness of the solution provided that $\mu \in L_\infty$ and $\sigma \in L_\infty$ are almost everywhere bounded from below by positive constants, $f \in (L_2(\Omega))^3$ and $\Omega \subset R^3$ is a bounded Lipschitz domain !

5. Derive appropriate mixed variational formulations of the Dirichlet BVP for the Poisson and the biharmonic equations, and discuss the advantages and the disadvantages of these mixed variational formulations against their primal variational formulations !

6. What do you know about mesh generation, regular meshes, and internal representation of the mesh with appropriate files ? Provide a mesh and its internal representation of some sample domain given by the examiner ! Give the definition of the FE Nodal Basis and of the V_h, V_{0h}, V_{gh} for a given triangular mesh via mapping technique !

7. Describe the generation of the FE equations $K_h \underline{u}_h = \underline{f}_h$ via the three steps

   a) assembling of the load vector,
   b) assembling of the stiffness matrix,
   c) incorporating the boundary conditions !

8. Describe the properties of the system of finite element equations and estimate the spectral condition number of the stiffness matrix K_h in the SPD case !

9. Prove the Bramble-Hilbert-Lemma and apply it to the proof of the approximation error estimate
$$\inf_{v_h \in V_h} |u - v_h|_{H^s(\Omega)} \leq ?$$
for $s = 0$ or $s = 1$, where $H^0(\Omega) = L_2(\Omega)$!

10. Prove the H^1-Convergence of the FE-solutions u_h to the solution u of a V_0-elliptic and V_0-bounded elliptic BVP (Theorem 2.8) !

11. How can you prove an optimal L_2 convergence rate estimate for the FE-solutions u_h to the solution u of a V_0-elliptic and V_0-bounded elliptic BVP (Theorem 2.10) !

12. What do you know about the L_∞-convergence of FE-solutions (Theorem 2.13) !

13. Prove the first Lemma of Strang ! What are the typical applications of the first Lemma of Strang ?

14. Prove the second Lemma of Strang ! What are the typical applications of the second Lemma of Strang ?

15. Explain the Clément interpolation operator and show its approximation properties (Lemma 2.18) !

16. Derive the residual error estimator for the homogeneous Dirichlet BVP for the Poisson equation (Theorem 2.19) ! How do you have to modify the residual error estimator in the case of mixed boundary conditions ?

17. Derive the residual error estimator for our heat conduction problem “CHIP” !

18. Construct a finite volume scheme for the second order BVP (1) given in Chapter 3 via the direct approximation of the balance equations !

19. Construct a finite volume scheme for the second order BVP (1) given in Chapter 3 via the Galerkin-Petrov Variational technique !

20. Describe the abstract approximation scheme for approximating operator equations $Au = b$ and prove the theorem on discrete and continuous convergence !

All Exercises given explicitly in lectures, i.e E1.1 - E1.11, E2.1 - E2.13, are also a subject of the Oral Examination !!!