T U T O R I A L

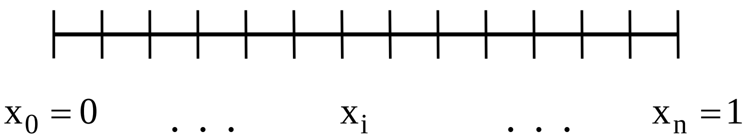
"Fast Solvers"

Tutorial 1

<u>01</u> Consider:

$$\begin{cases} -u''(x) = f(x), \quad x \in (0,1) \\ u(0) = u(1) = 0 \end{cases}$$

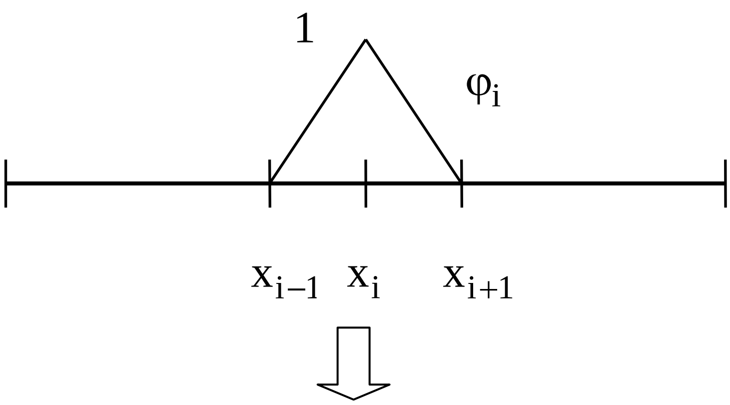
Grid (natural numeration of nodes):



$$h = 1/n$$

$$x_i = i \cdot h, \quad i = 0, 1, ..., n$$

FE: $\left\{ \varphi_i \right\}_{i=1}^{n=1}$



Define the system

$$A\bar{u} = \bar{f}$$

Consider

$$A\bar{q}_i = \lambda_i \bar{q}_i$$

<u>Prove</u>

$$\bar{q}_i = \begin{bmatrix} q_i(1) \\ \cdots \\ q_i(j) \\ \cdots \\ q_i(n-1) \end{bmatrix}, \quad q_i(j) = \sqrt{2/n}\,\sin\frac{i\pi j}{n}$$

<u>Find</u>

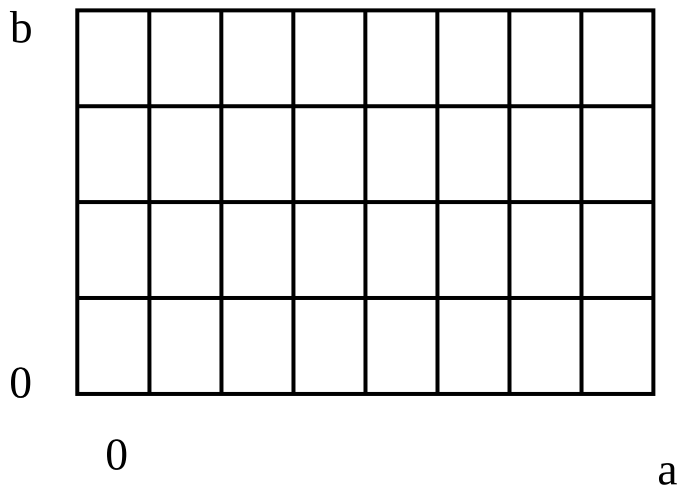
$$\lambda_i = ?$$

<u>02</u> Consider:

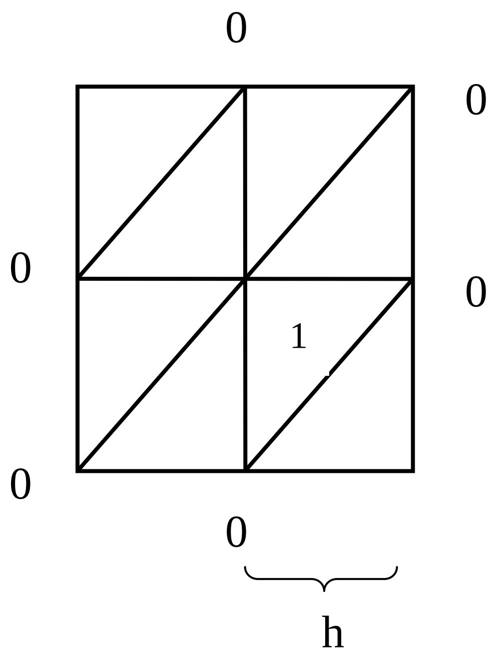
$$\begin{cases} -\Delta u \equiv -(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) = f(x), & x \in \Omega \equiv (0,a)\times(0,b) \\ u(x) = 0, & x \in \partial\Omega \end{cases}$$

Let h be a mesh step,

$$a = h \cdot n, \quad b = h \cdot m$$



F.E.:



a) <u>Define</u> the numeration of nodes

b) <u>Define</u> the system

$$A\bar{u} = \bar{f}$$

c) <u>Find</u> eigenvectors and eigenvalues of A

<u>03</u> Let A be SPD matrix

$$A = A^* > 0$$

Prove

$$\lambda_{\min}(A) = \inf_{\bar{u}\neq 0} \frac{(A\bar{u},\bar{u})}{(\bar{u},\bar{u})},$$

$$\lambda_{\max}(A) = \sup_{\bar{u}\neq 0} \frac{(A\bar{u},\bar{u})}{(\bar{u},\bar{u})},$$

where

$\lambda_{\min}(A)$ is the minimum eigenvalue,

$\lambda_{\max}(A)$ is the maximum eigenvalue.

04 Give an asymptotic estimate with respect to a mesh step h of

$$\mathrm{cond}(A) = \frac{\lambda_{\max}(A)}{\lambda_{\min}(A)} \quad \text{for A from } \underline{01},\ \underline{02}.$$

05 Let

$$A = A^* > 0,$$

$$B = B^* > 0,$$

$$c_1(B\bar{u},\bar{u}) \le (A\bar{u},\bar{u}) \le c_2(B\bar{u},\bar{u}) \quad \forall\bar{u}.$$

Estimate

$$\mathrm{cond}(B^{-1/2} A B^{-1/2}) \le ?$$