T U T O R I A L

"Fast Solvers"

Tutorial 2

<u>06</u> Let

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

such that

$$\det(A) \neq 0, \quad \det(A_{11}) \neq 0$$

and let

$$A^{-1} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}.$$

<u>Prove</u>

$$S_A^{-1} = B_{22},$$

where

$$S_A = A_{22} - A_{21} A_{11}^{-1} A_{12}$$

07 Let

$$\Omega = \{(x_1, x_2) \mid 0 < x_1 < 1,\ \ 0 < x_2 < 1\},$$

$$\Gamma_1 = \{(x_1, 0) \mid 0 < x_1 < 1\},$$

$$\Gamma_0 = \partial\Omega \setminus \Gamma_1.$$

Consider

$$\begin{cases} -\Delta u = f(x), & \text{in } \Omega, \\ u(x) = 0, & x \in \Gamma_0, \\ \dfrac{\partial u(x)}{\partial n} = 0, & x \in \Gamma_1. \end{cases}$$

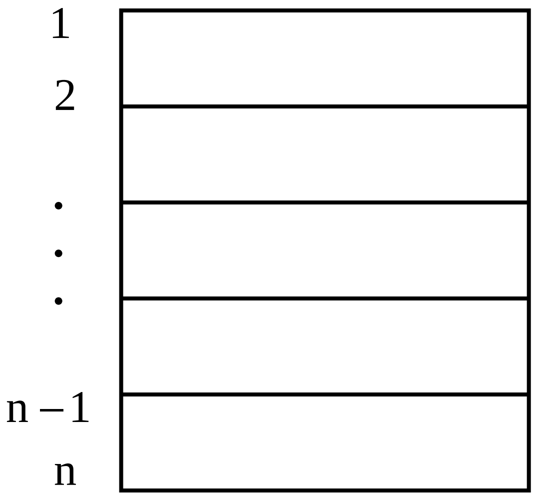
Let Ω^h be a uniform triangulation with a mesh step $h = 1/n$ and $H_h(\Omega^h, \Gamma_0)$ be a piecewise linear finite-element space such that

$$u^h(x) = 0,\ \ x \in \Gamma_0.$$

Define the stiffness matrix A:

$$\int_\Omega (\nabla u^h, \nabla v^h) = (Au, v) \quad \forall u^h, v^h \in H_h(\Omega^h, \Gamma_0).$$

Consider the decomposition of nodes of Ω^h into blocks with the numeration of blocks as on the picture:



Let A_1 is the matrix of the order n-1 having the following form

$$A_1 = \begin{bmatrix} 2 & -1 & & & & & \\ -1 & 2 & -1 & & & & \\ & \cdot & \cdot & \cdot & & & \\ & & \cdot & \cdot & \cdot & & \\ & & & \cdot & \cdot & \cdot & \\ & & & & -1 & 2 & -1 \\ & & & & & -1 & 2 \end{bmatrix}$$

and let

$$A_1 = Q_1 \Lambda_1 Q_1^h ,$$

$$Q_1 = [q_1, \dots, q_{n-1}], \quad \Lambda_1 = \mathrm{diag}\{\lambda_1, \dots, \lambda_{n-1}\},$$

where $\{q_i\}$, $\{\lambda_i\}$ are eigenvectors and eigenvalues of A_1.

a) Find the representation of A in the form

$$A = QTQ^*,$$

where Q is the block-diagonal matrix:

$$Q = \begin{bmatrix} Q_1 & & & \\ & \cdot & & \\ & & \cdot & \\ & & & \cdot \\ & & & & Q_1 \end{bmatrix} \begin{matrix} \uparrow \\ | \\ | \\ | \\ \downarrow \end{matrix} \; n-1.$$

b) For each $u^h \in H_h(\Omega^h, \Gamma_0)$ consider the corresponding vector $u \in R^{(n-1)n}$. Decompose a vector u into blocks

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix},$$

where the vector u_1 corresponds to nodes from the interior of Ω and the vector u_2 corresponds to nodes from Γ_1. According to this decomposition, A has the block form

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}.$$

Put

$$S_A = A_{22} - A_{21} A_{11}^{-1} A_{12}.$$

Find eigenvectors of S_A.

c) * Find a recurrent formula to define eigenvalues of S_A.