

• Remark:

1. First BVP, i.e. $\Gamma_u = \Gamma$:

In the case $H_{0,\Omega}^1(\Omega) = (H^1(\Omega))^3$, the second equation of (24) gives, for $q = 1 \in L_2(\Omega)$:

$$\frac{1}{\lambda}(p, 1) = (\operatorname{div} u, 1) = \int_{\Omega} \operatorname{div} u \, dx = -0 + \int_{\Gamma} u \cdot n \, ds = 0, \text{ i.e.}$$

(25) $\int_{\Omega} p \, dx = 0$ in the case of the 1st BVP.

Therefore, $\Lambda_0 = \{p \in L_2(\Omega) : \int_{\Omega} p \, dx = (p, 1)_0 = 0\} = L_2(\Omega) / \mathbb{R}$

$$\text{with } \|p\|_{\Lambda_0} := \|p\|_0 := \inf_{c \in \mathbb{R}} \|p + c\|_0$$

2. Second BVP, i.e. $\Gamma_u = \emptyset$:

$$H_{0,\Omega}^1(\Omega) = \{v \in (H^1(\Omega))^3 : (v, r)_0 = 0 \, \forall r \in \mathcal{R}\} = \tilde{H}^1(\Omega)$$

↑
rigid body motions

• (24) can be written as abstract mixed variational problem (2.34) with perturbation terms

(24) Find $u \in X = H_{0,\Omega}^1(\Omega)$ and $p \in \Lambda = \Lambda_0 = L_2(\Omega)$:

$$a(u, v) + b(v, p) = \langle F, v \rangle \quad \forall v \in X,$$

$$b(u, q) - t^2 c(p, q) = \langle G, q \rangle \quad \forall q \in \Lambda,$$

with

$$a(u, v) := 2\mu (\varepsilon(u), \varepsilon(v))_0,$$

$$b(v, p) := (\operatorname{div} v, p)_0,$$

$$c(p, q) := (p, q)_0, \quad t^2 = 1/\lambda = \frac{1+\nu}{E} (1-2\nu),$$

$$\langle F, v \rangle := \int_{\Omega} f^T v \, dx + \int_{\Gamma} t^T v \, ds - \alpha \int_{\Omega} \gamma v \, dx,$$

$$\langle G, q \rangle := 0 \quad \text{with } \gamma = \frac{1}{2} \operatorname{tr} \varepsilon(u) \text{ is the dilatation}$$